

# Decomposing Images into Layers via RGB-space Geometry

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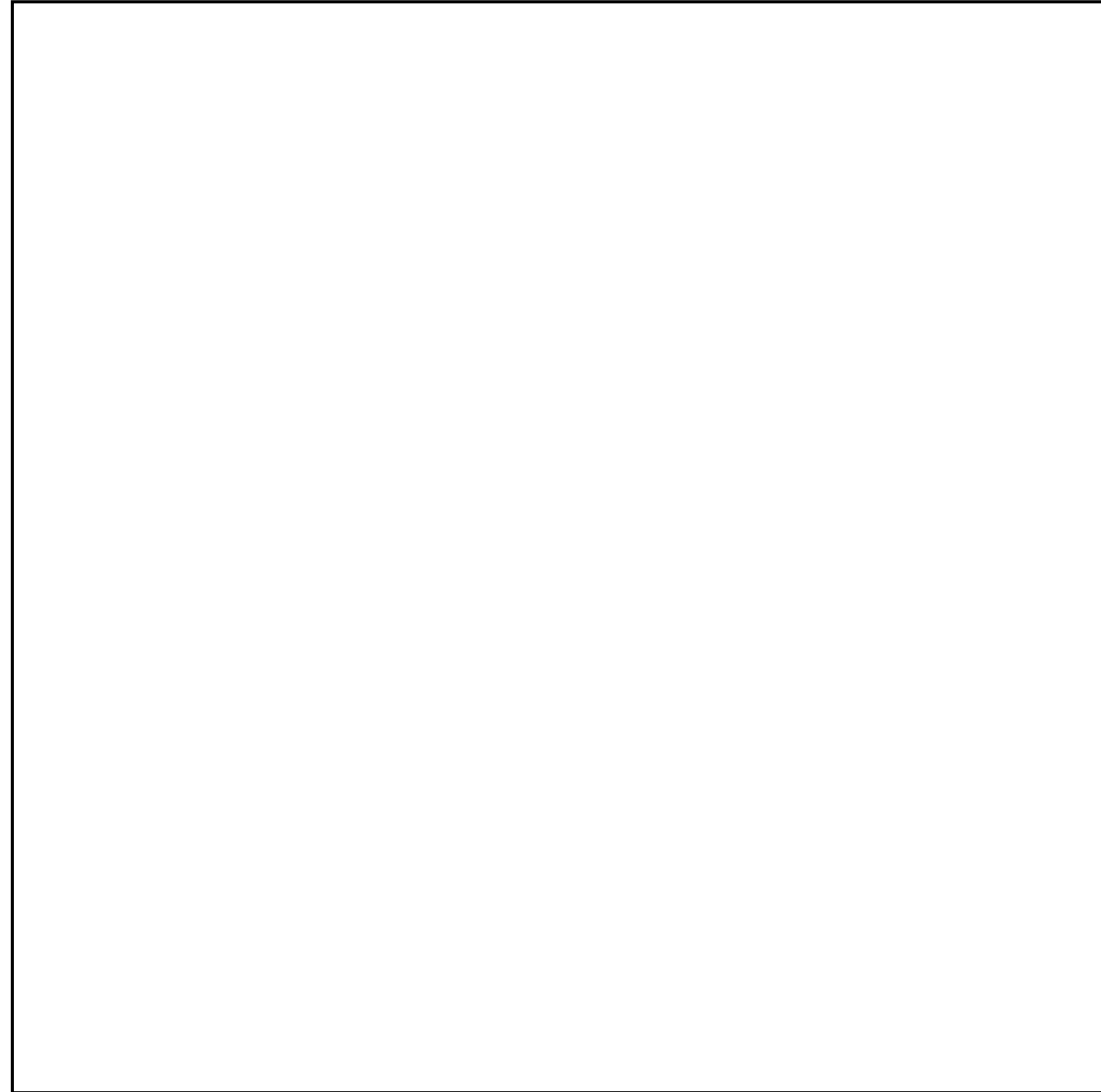
**CraGL**  
Creativity and Graphics Lab

 **GEORGE  
MASON**  
UNIVERSITY

**MASC**  
Motion and Shape Computing

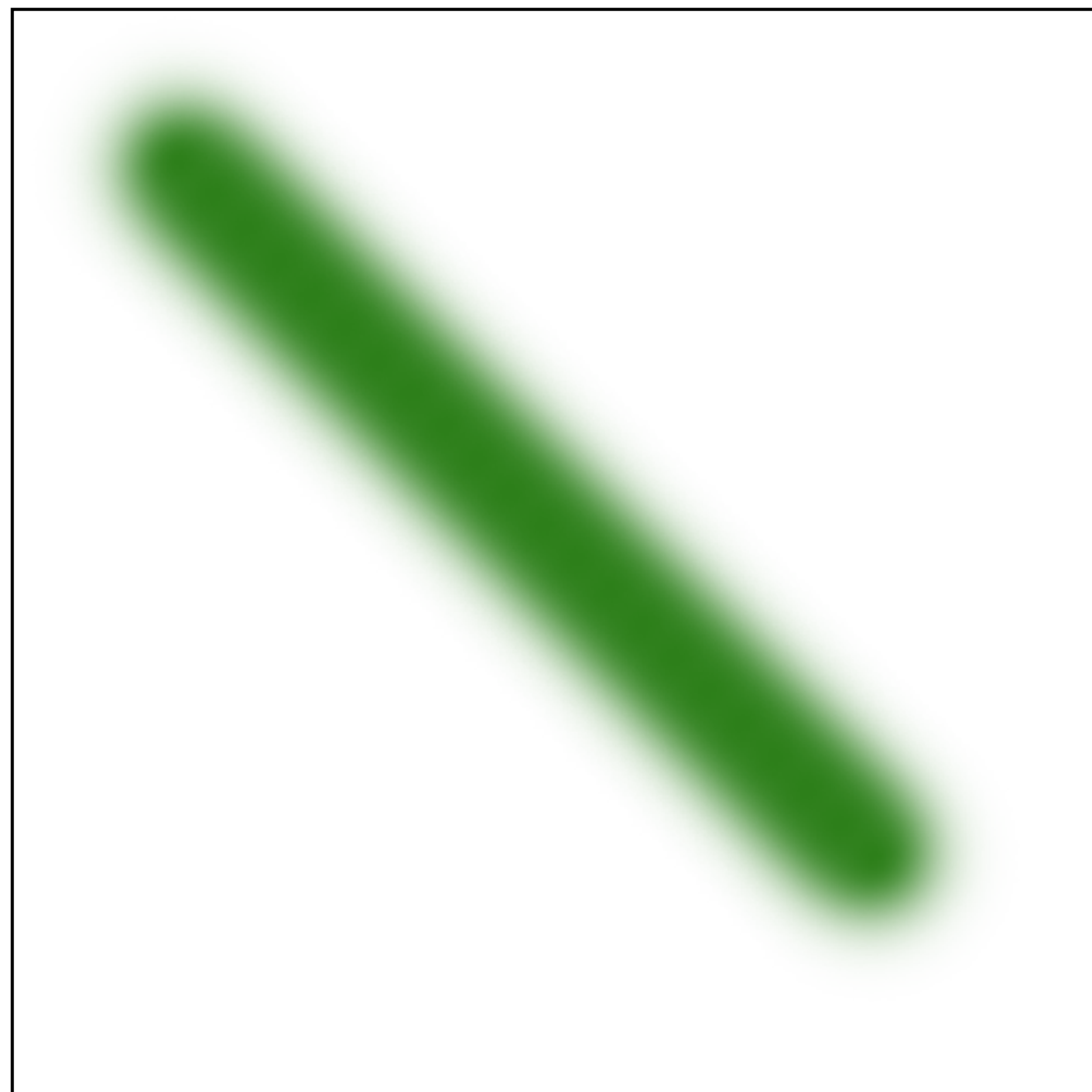


# Background: Digital Painting





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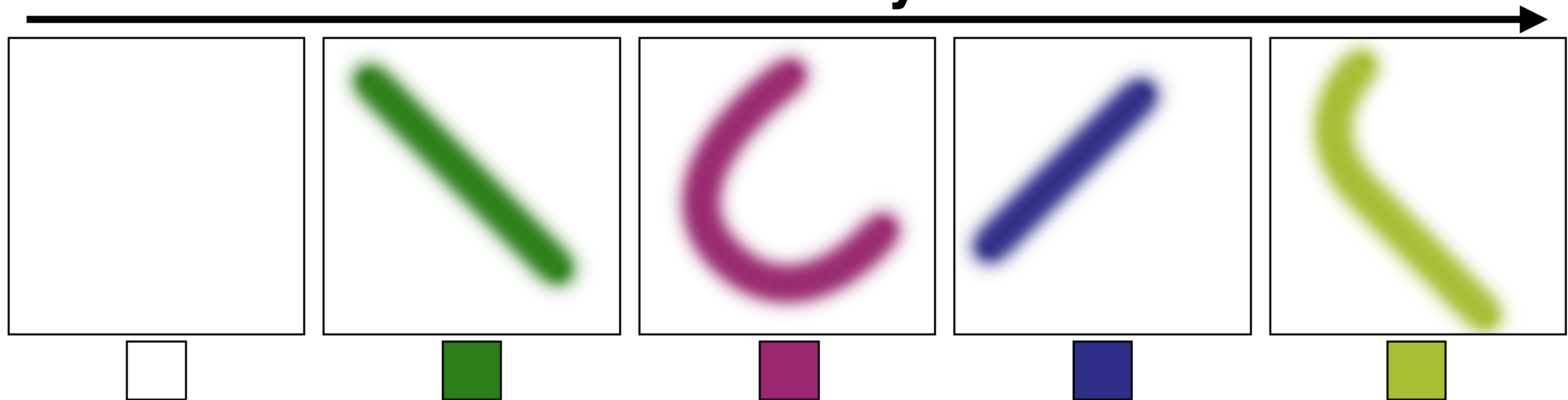


# Background: Digital Painting



# Background: Digital Painting

## Ordered Layers



# Background: Digital Painting





# Motivation: Layers Organize Images

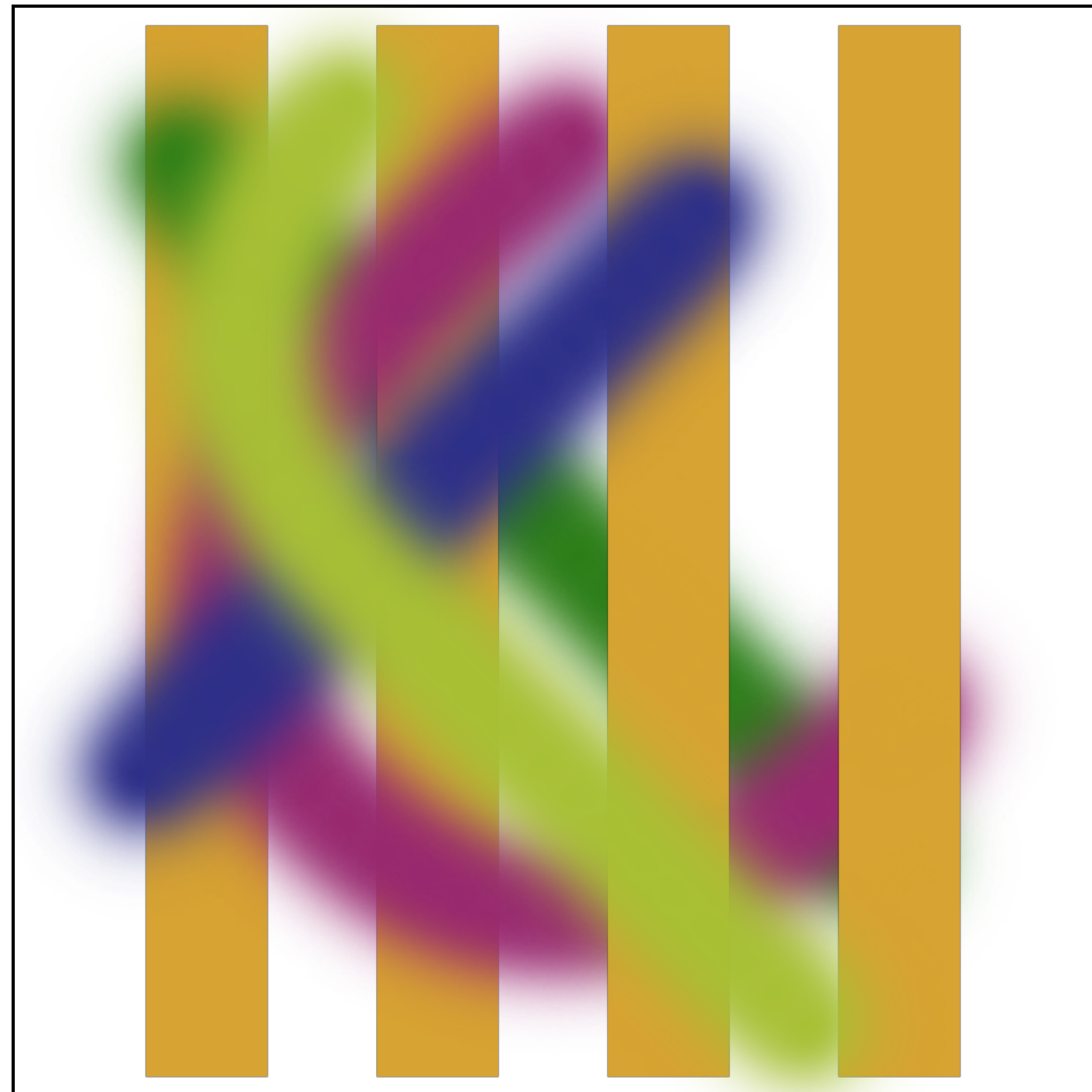


# Motivation: Layers Organize Images





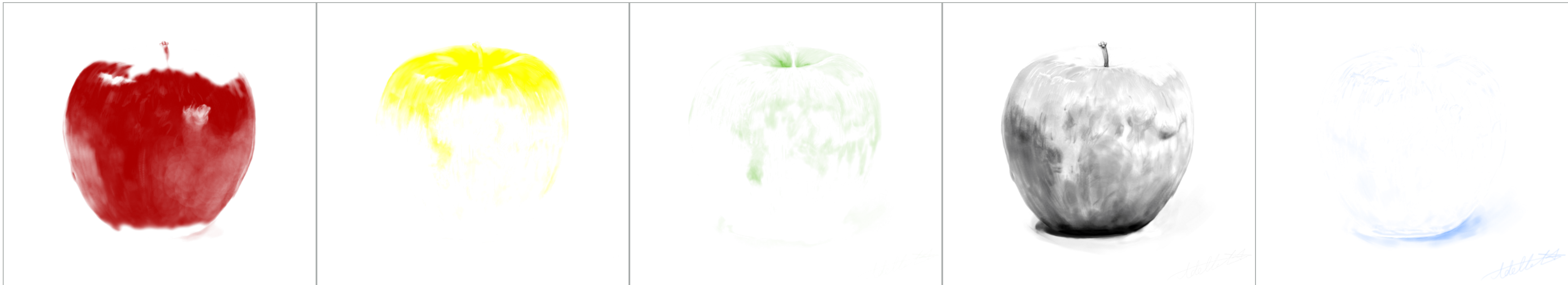
# Motivation: Layers Organize Images



# Images in the wild don't have layers



# Can we decompose them into layers?





That reproduce the original image?



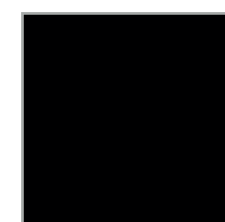
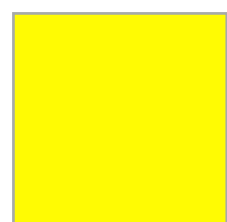
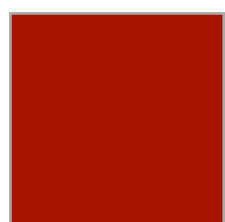
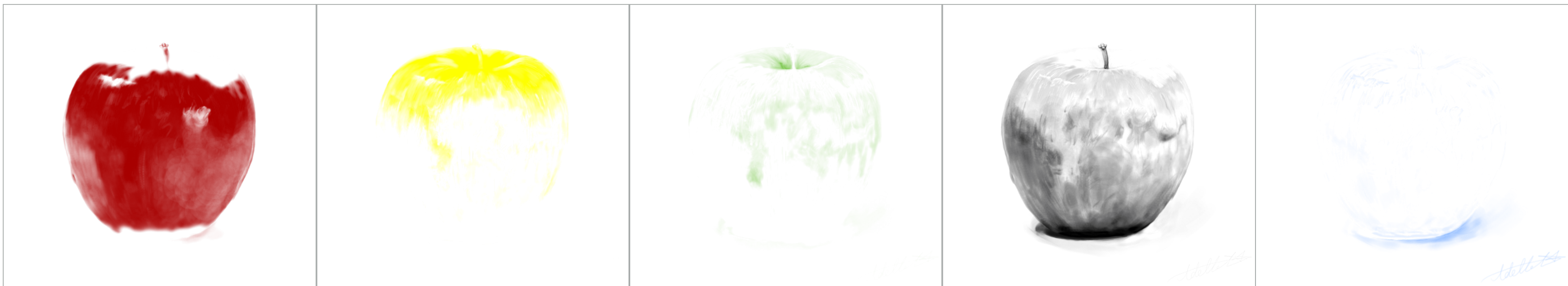
# Two Subproblems



# Layer Colors (Coats of Paint)



# Layer Opacity





# Related Work

- Interacting with editing history
  - Su et al. [2009], VisTrails [2009], McCann and Pollard [2009; 2012], Grossman et al. [2010], Noris et al. [2012], Denning and Pellacini [2013], Chen et al. [2014], Matzen and Snavely [2014], Karsch et al. [2014]. Amati and Brostow [2010], Hu et al. [2013]. Tan et al. [2015].

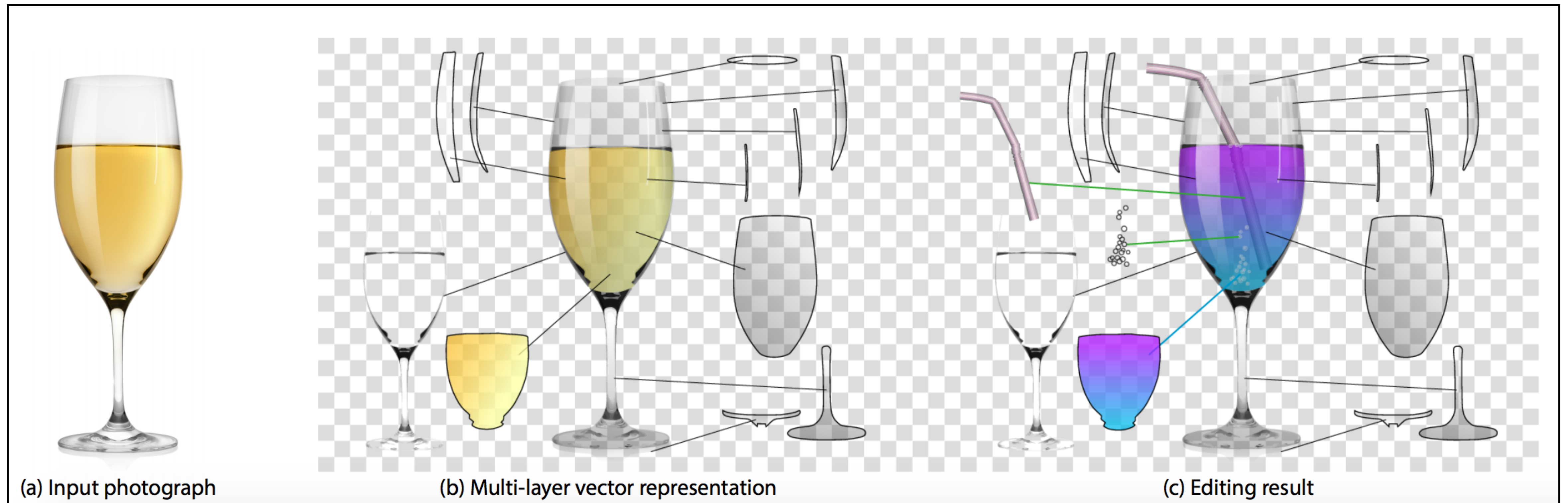


Decomposing time-lapse paintings into layers [Tan et al. 2015]



# Related Work

- Decomposing edits
  - Xu et al. [2006], Amati and Brostow [2010], Fu et al. [2011], Hu et al. [2013], Richardt et al. [2014].



Vectorising bitmaps into semi-transparent gradient layers [Richardt et al. 2014]

# Related Work

- Image matting
  - Smith and Blinn [1996], Zongker et al. [1999], Farid and Adelson [1999], Szeliski et al. [2000], Levin et al. [2006; 2008] and so on.



Blue screen matting [Smith and Blinn 1996]



# Related Work

- Palette Selection

- Shapira et al. [2009], O'Donovan et al. [2011], Lin et al. [2013], Gerstner et al. [2013], Chang et al. [2015].

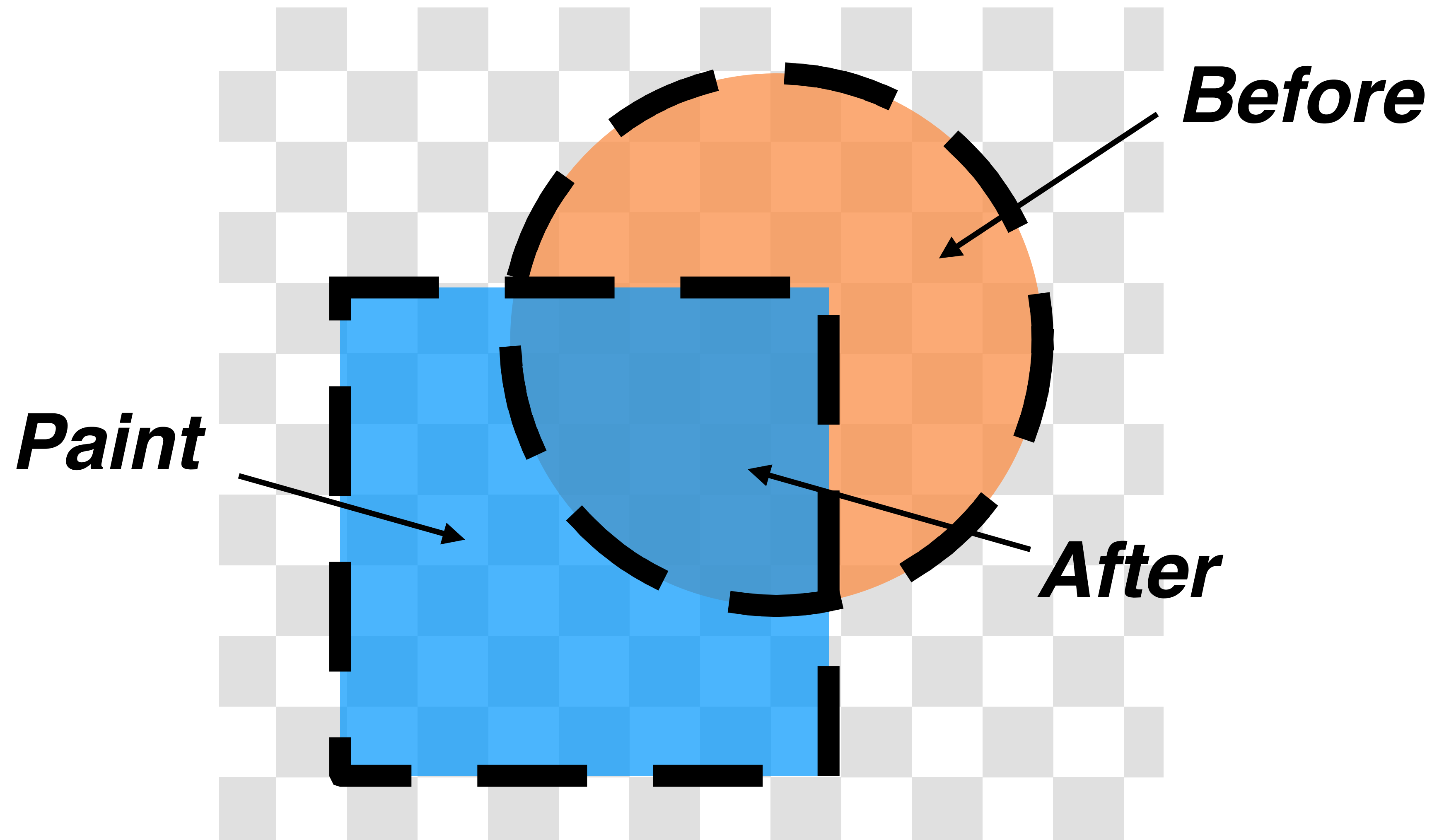


Palette based photo recoloring [Chang et al. 2015]

# Geometry of Compositing

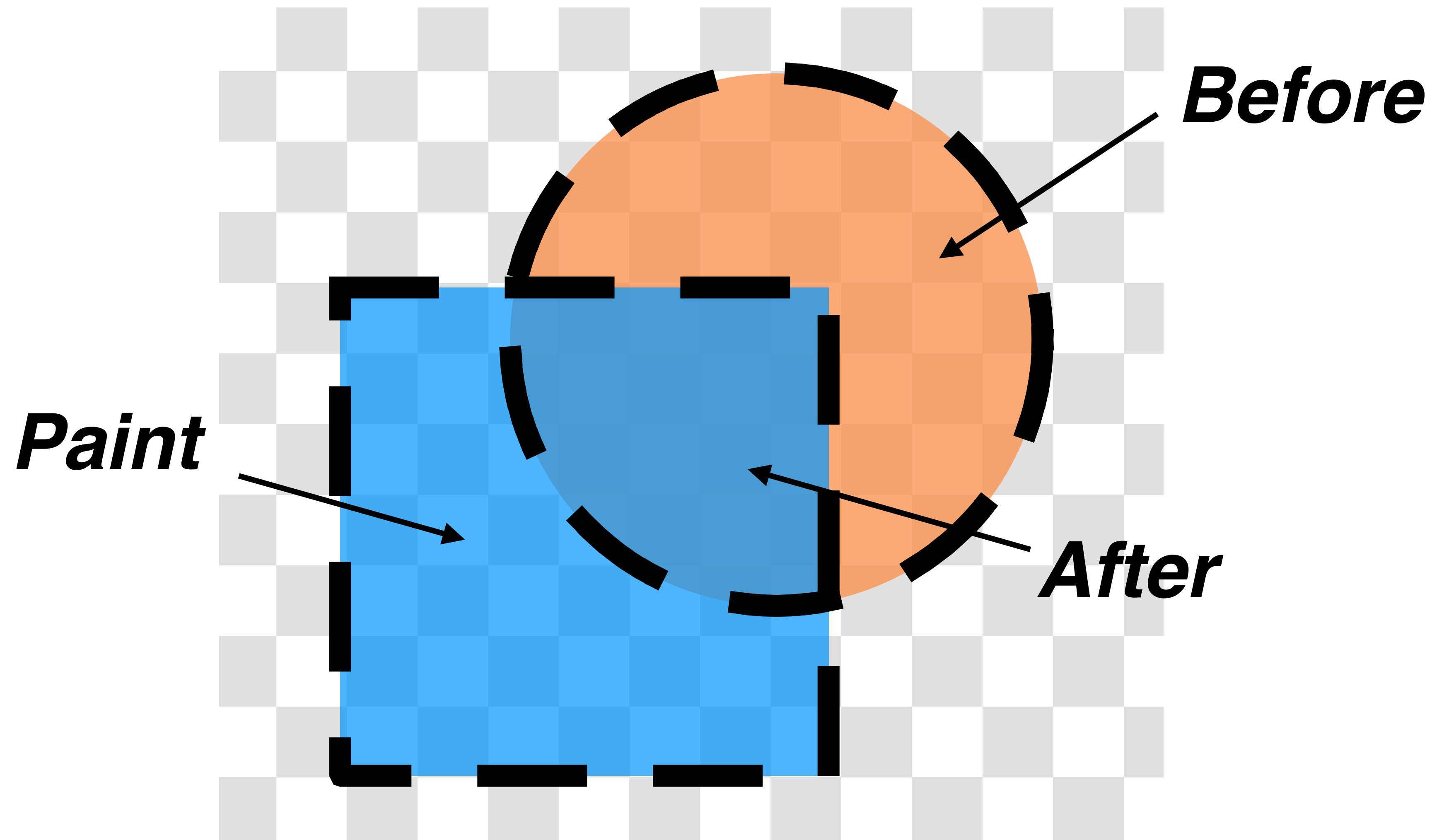
# Porter-Duff “Over” Color Compositing

$$After = Before \cdot (1 - \alpha) + Paint \cdot \alpha$$

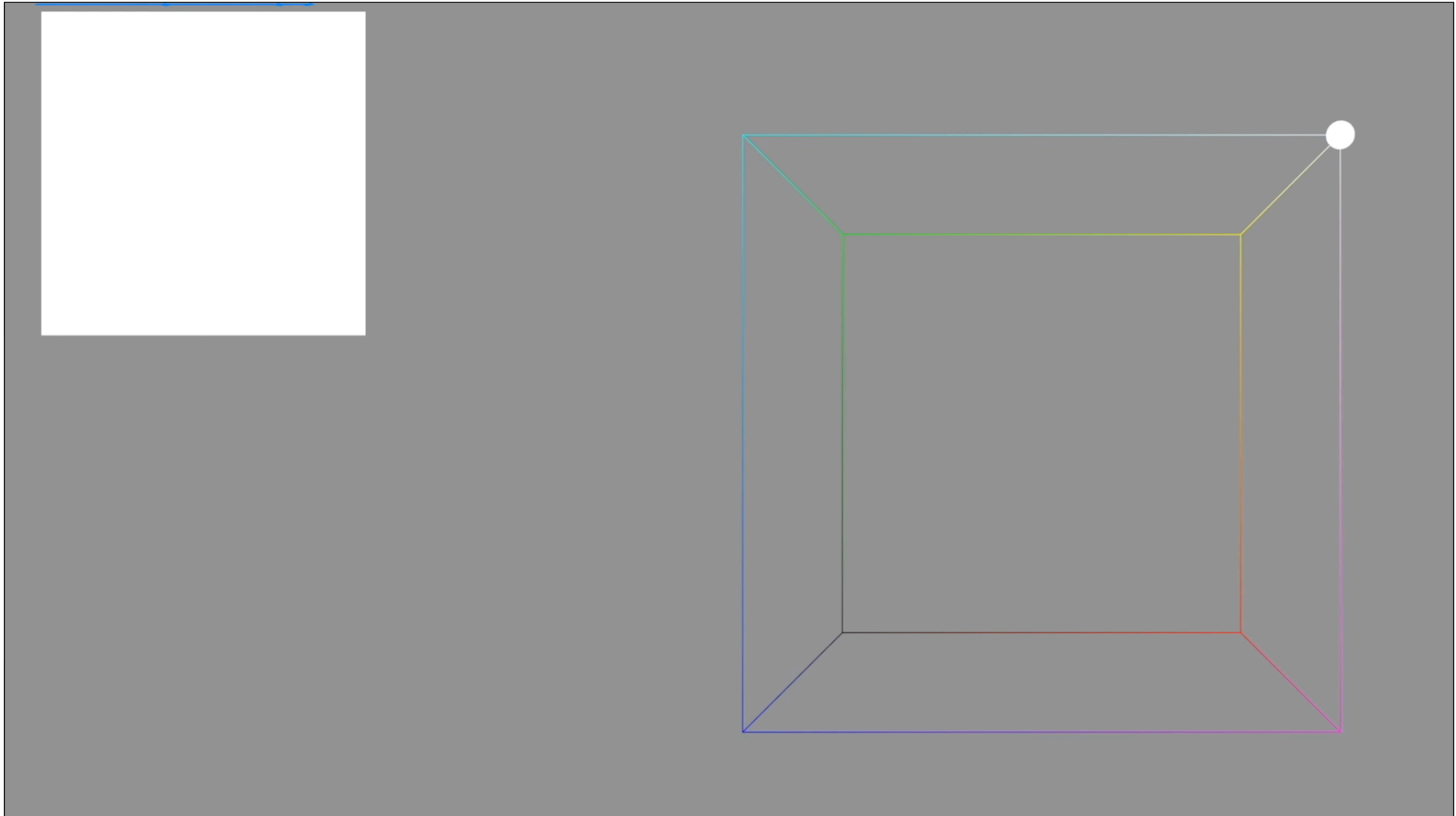


# Porter-Duff “Over” Color Compositing

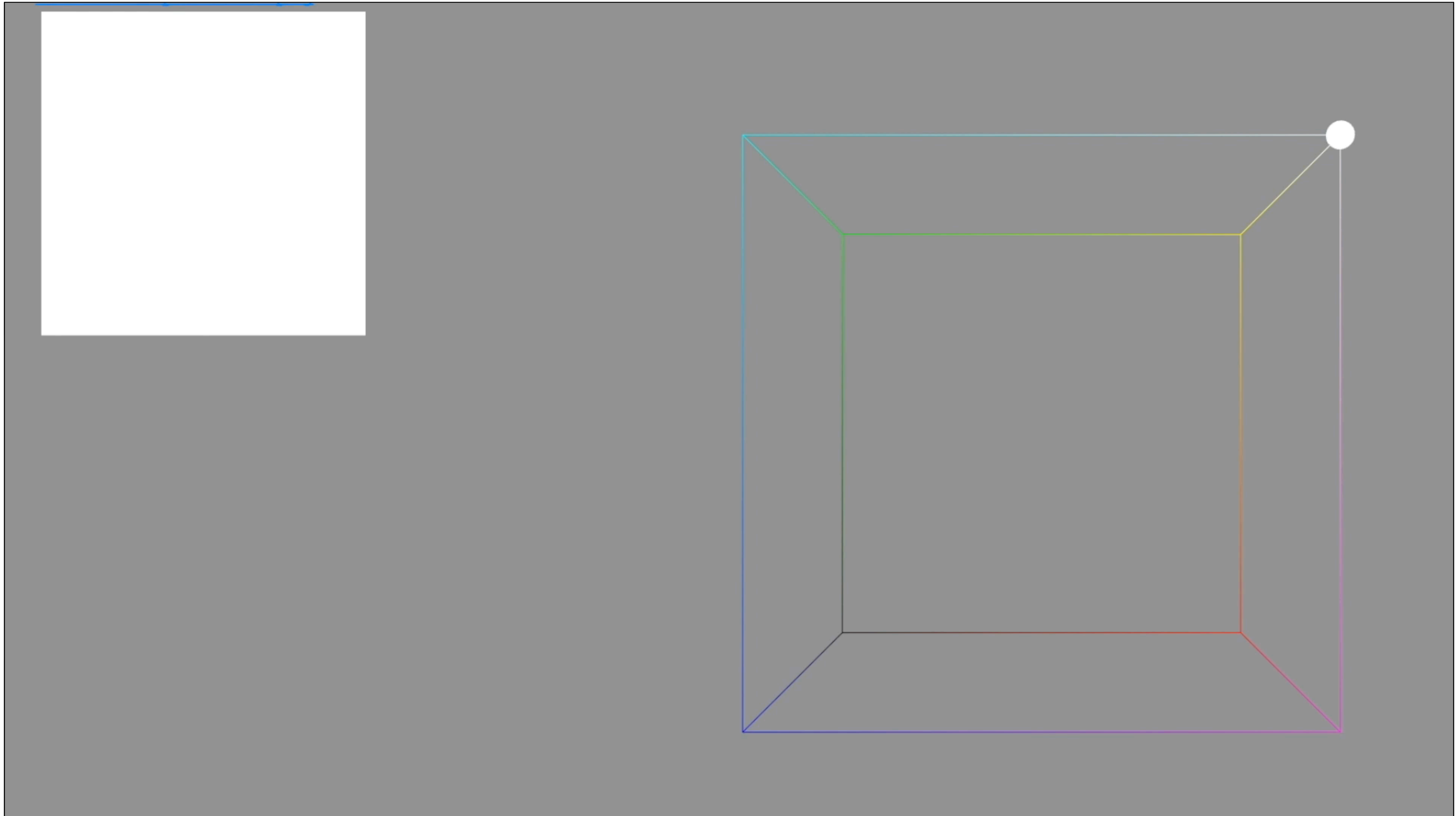
$$After = Before \cdot (1 - \alpha) + Paint \cdot \alpha$$



Coats of Paint follow a convex structure in RGB space

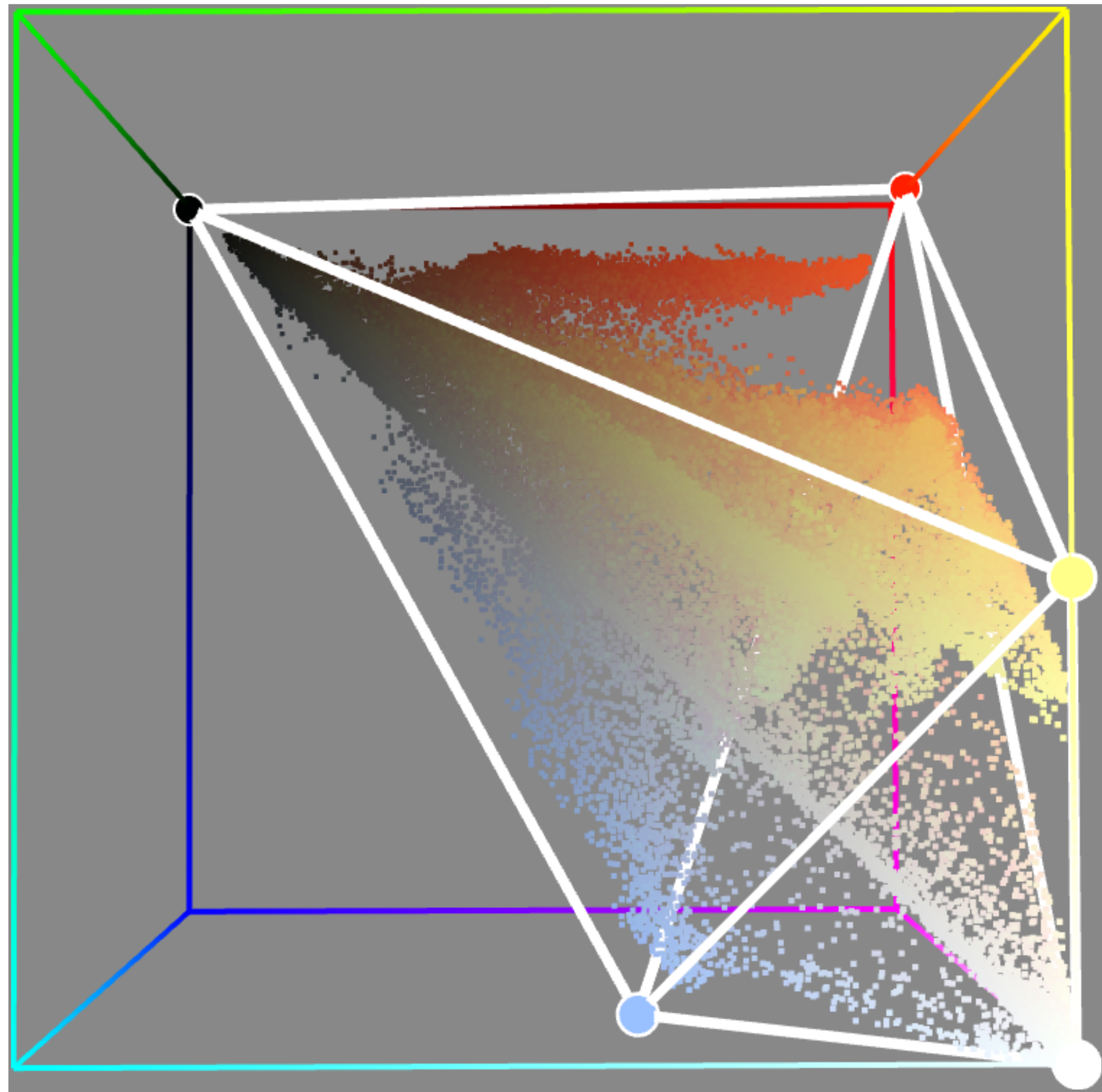


Coats of Paint follow a convex structure in RGB space



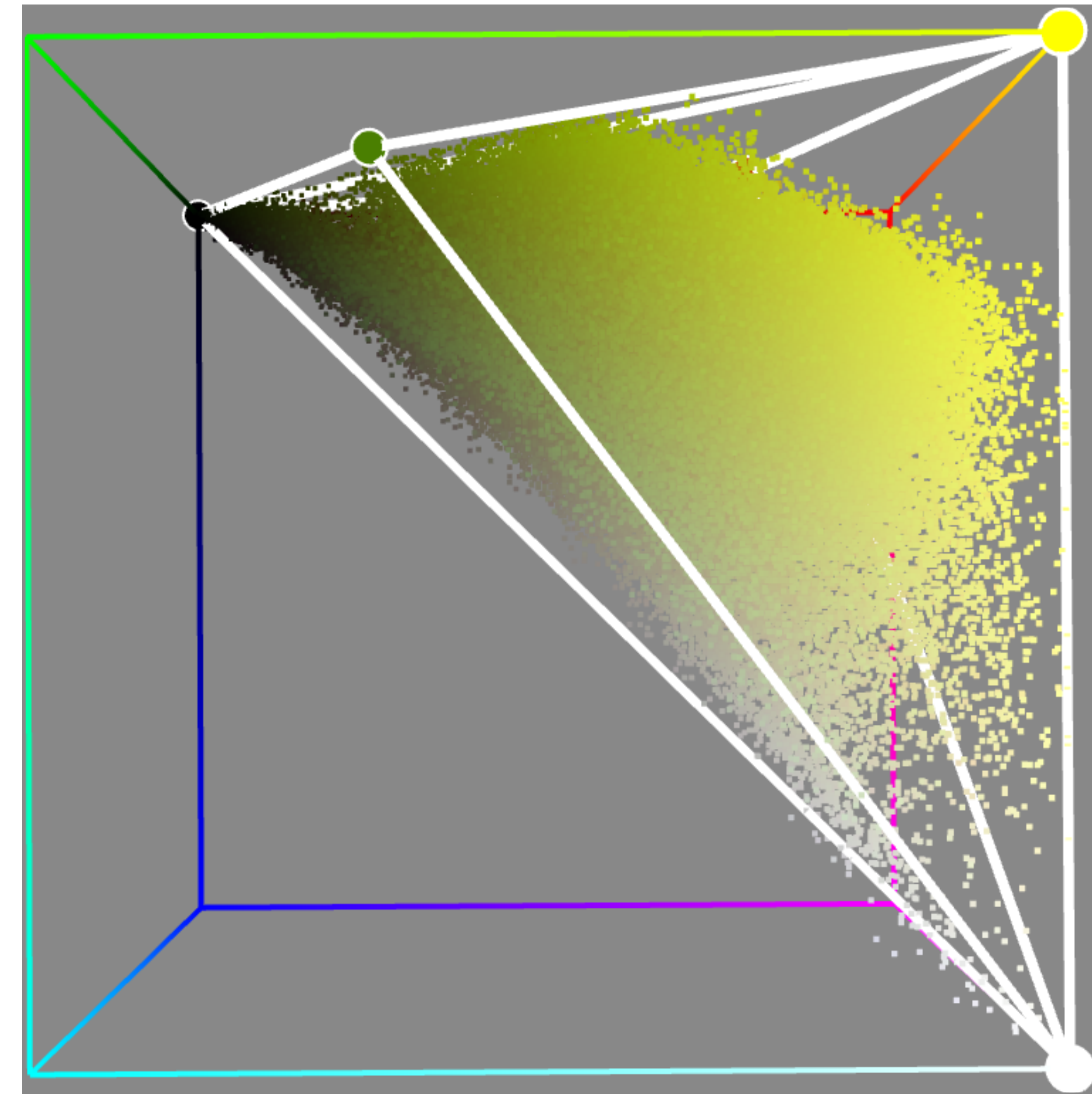


# More examples



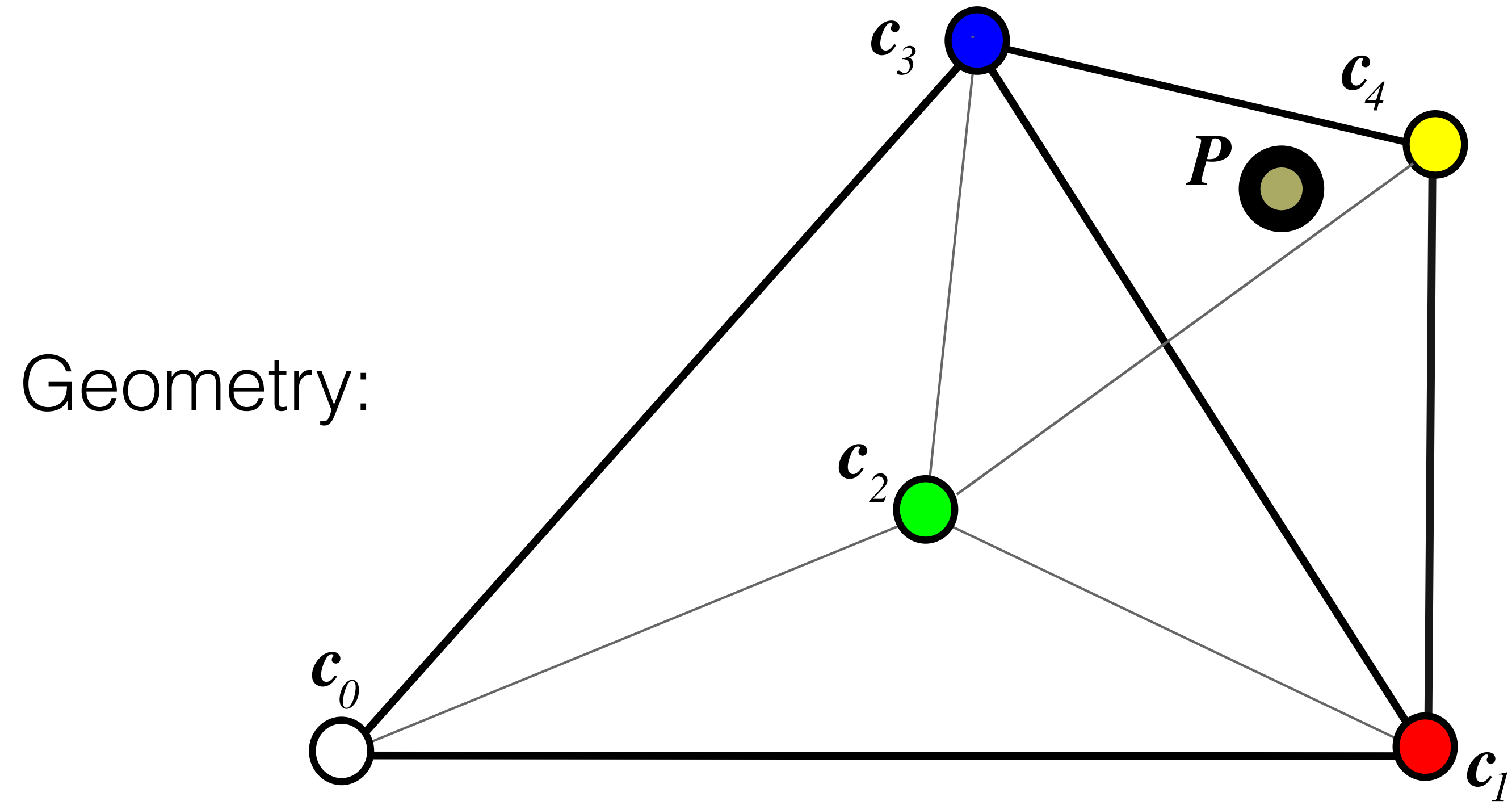


# More examples

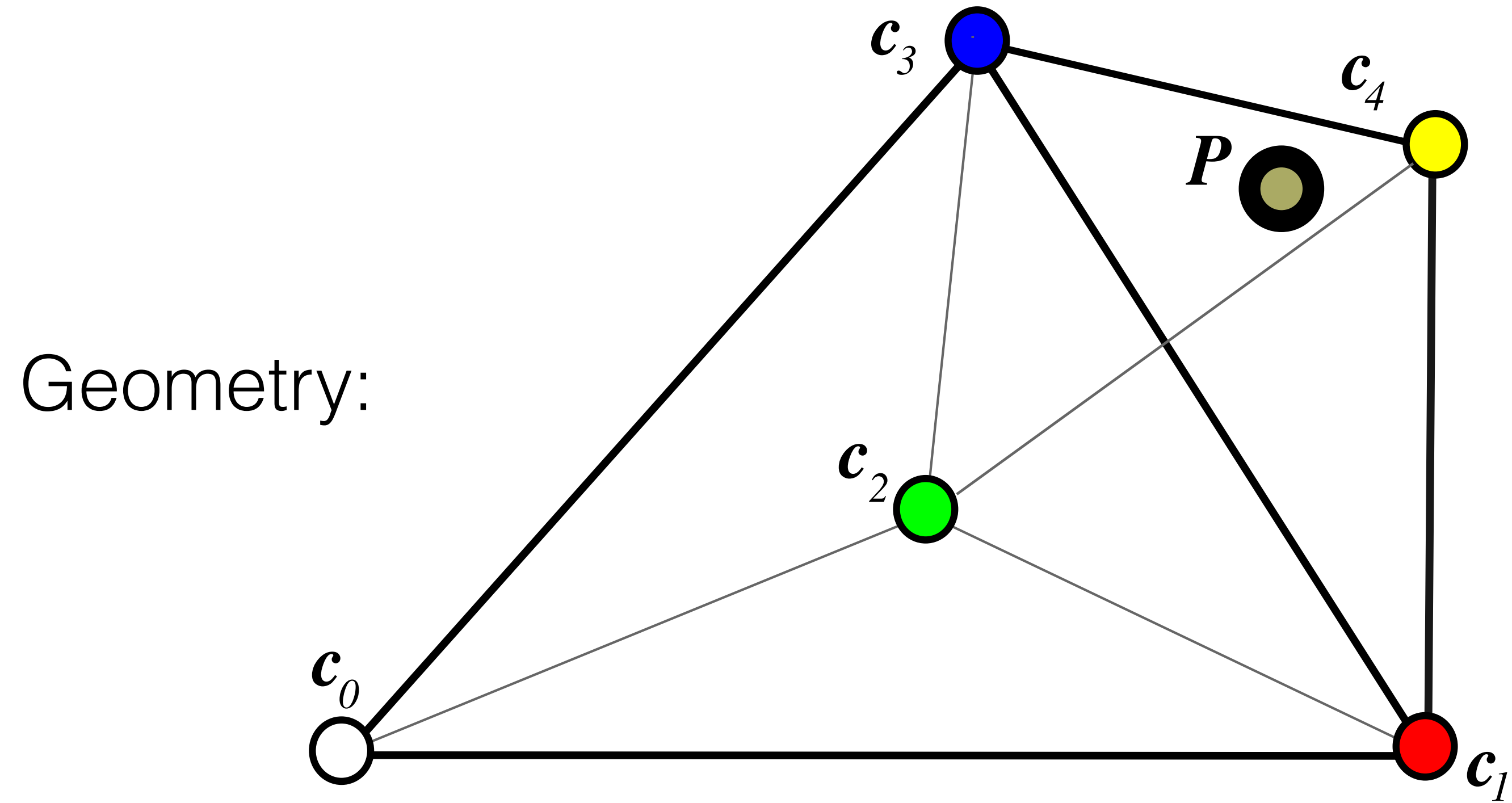




# Geometric interpretation of 'over' compositing equation



# Geometric interpretation of 'over' compositing equation



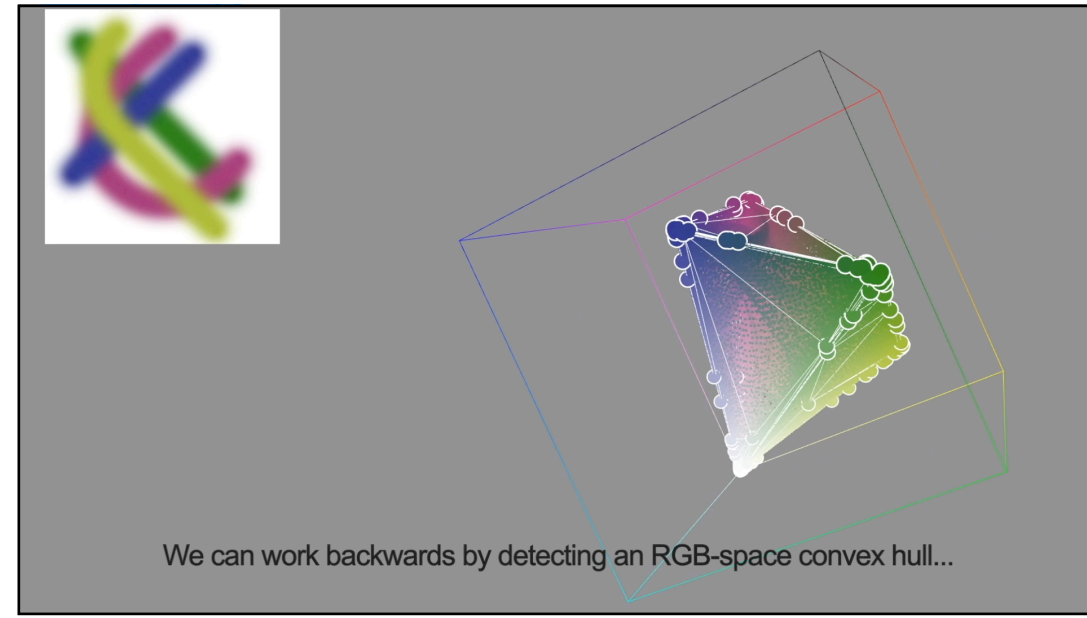
Algebra:

$$\mathbf{p} = \mathbf{c}_n + \sum_{i=1}^n \left[ (\mathbf{c}_{i-1} - \mathbf{c}_i) \prod_{j=i}^n (1 - \alpha_j) \right]$$

# Our Pipeline



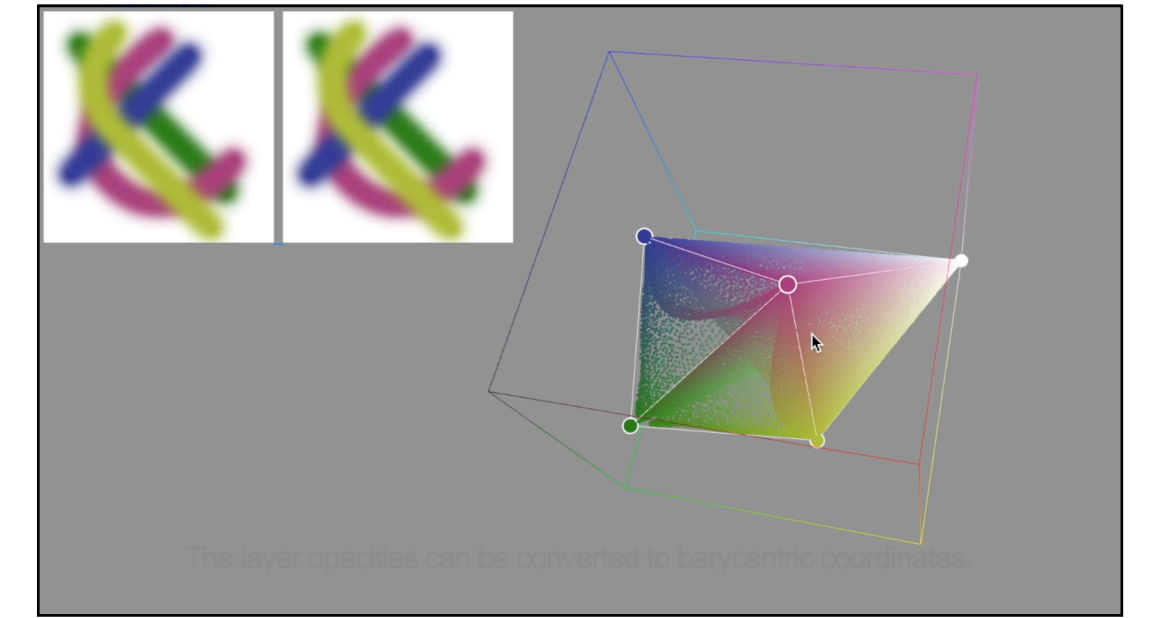
Input



Palette selection



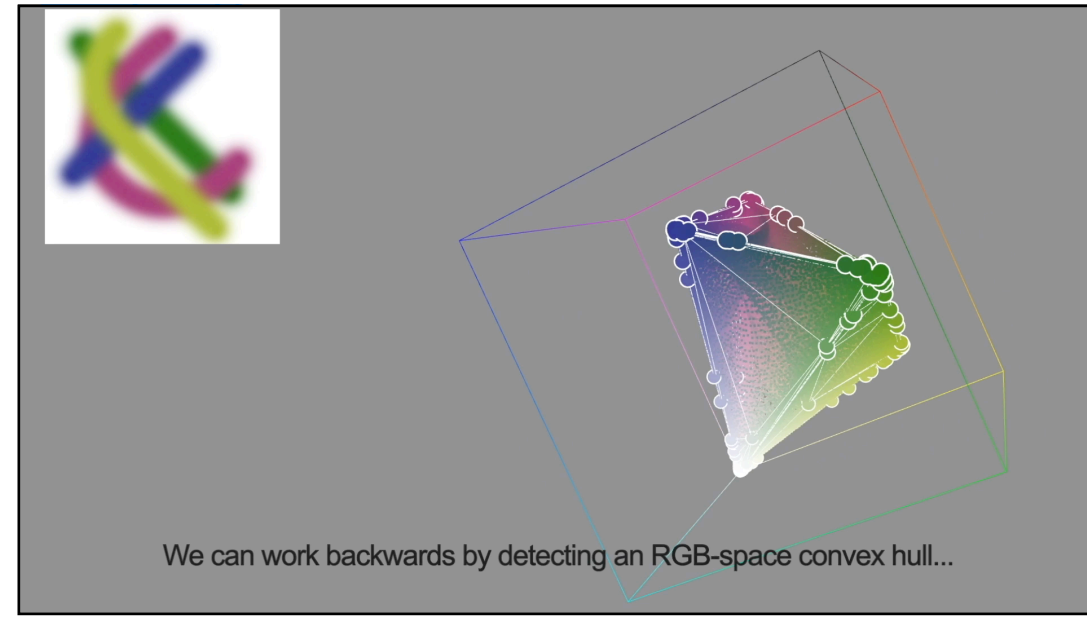
Layer opacity



Edit



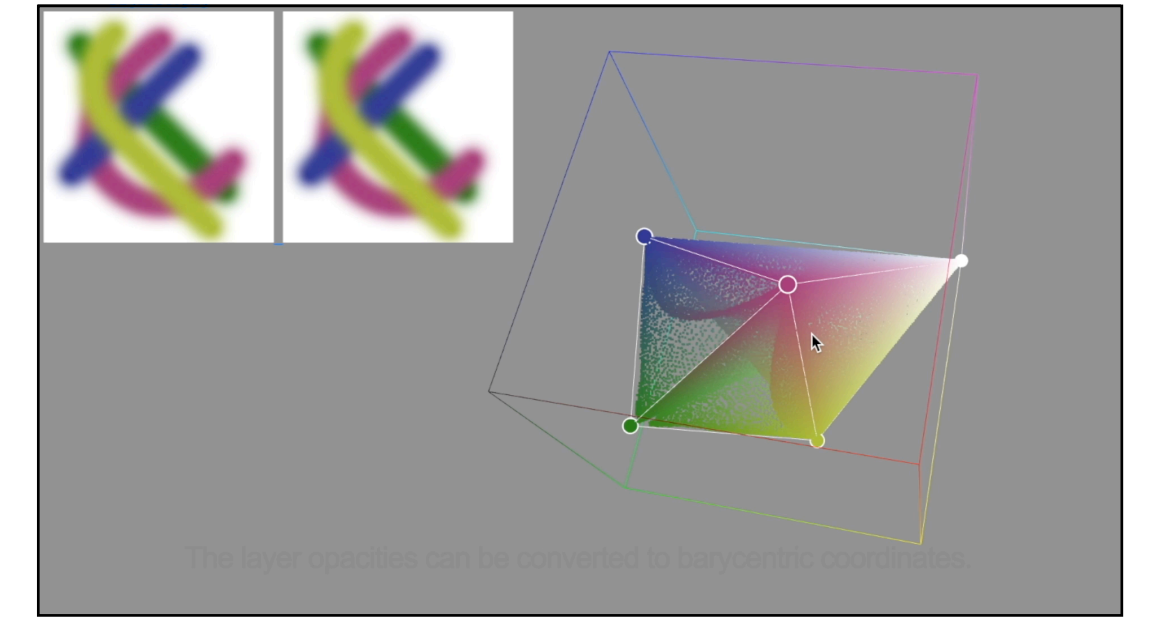
Input



Palette selection



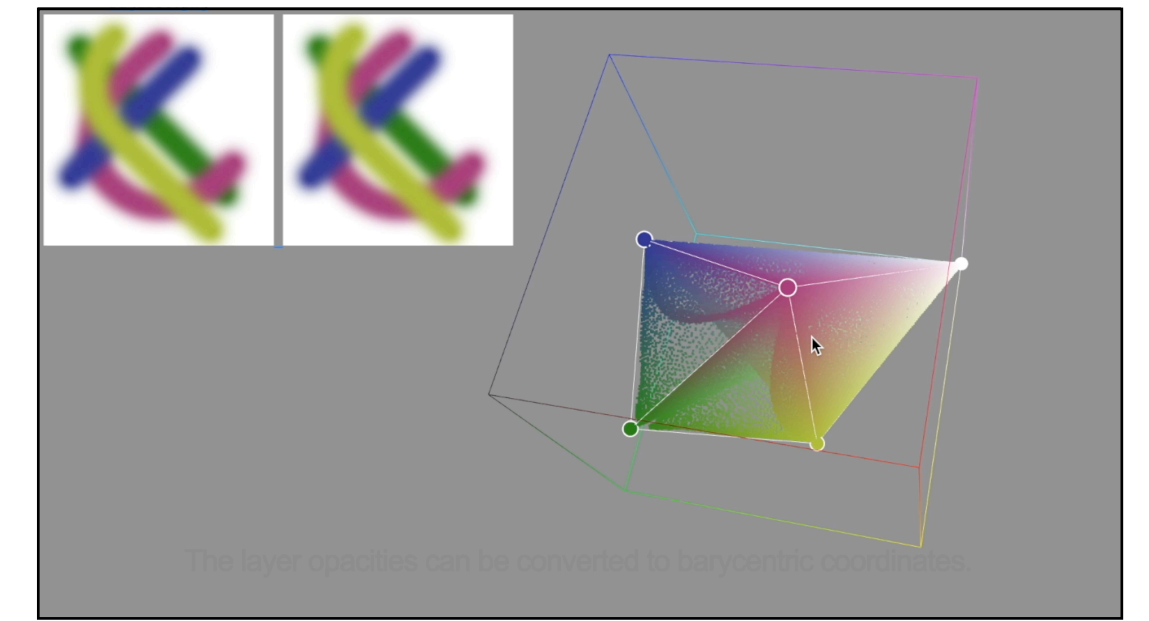
Layer opacity



Edit



Input

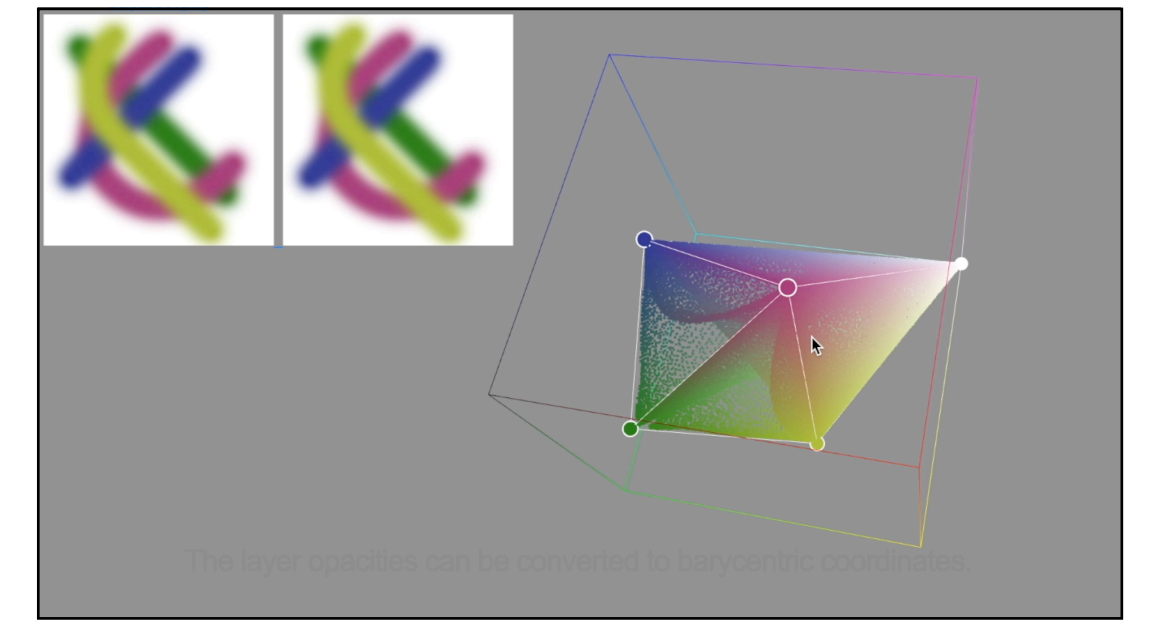


Edit

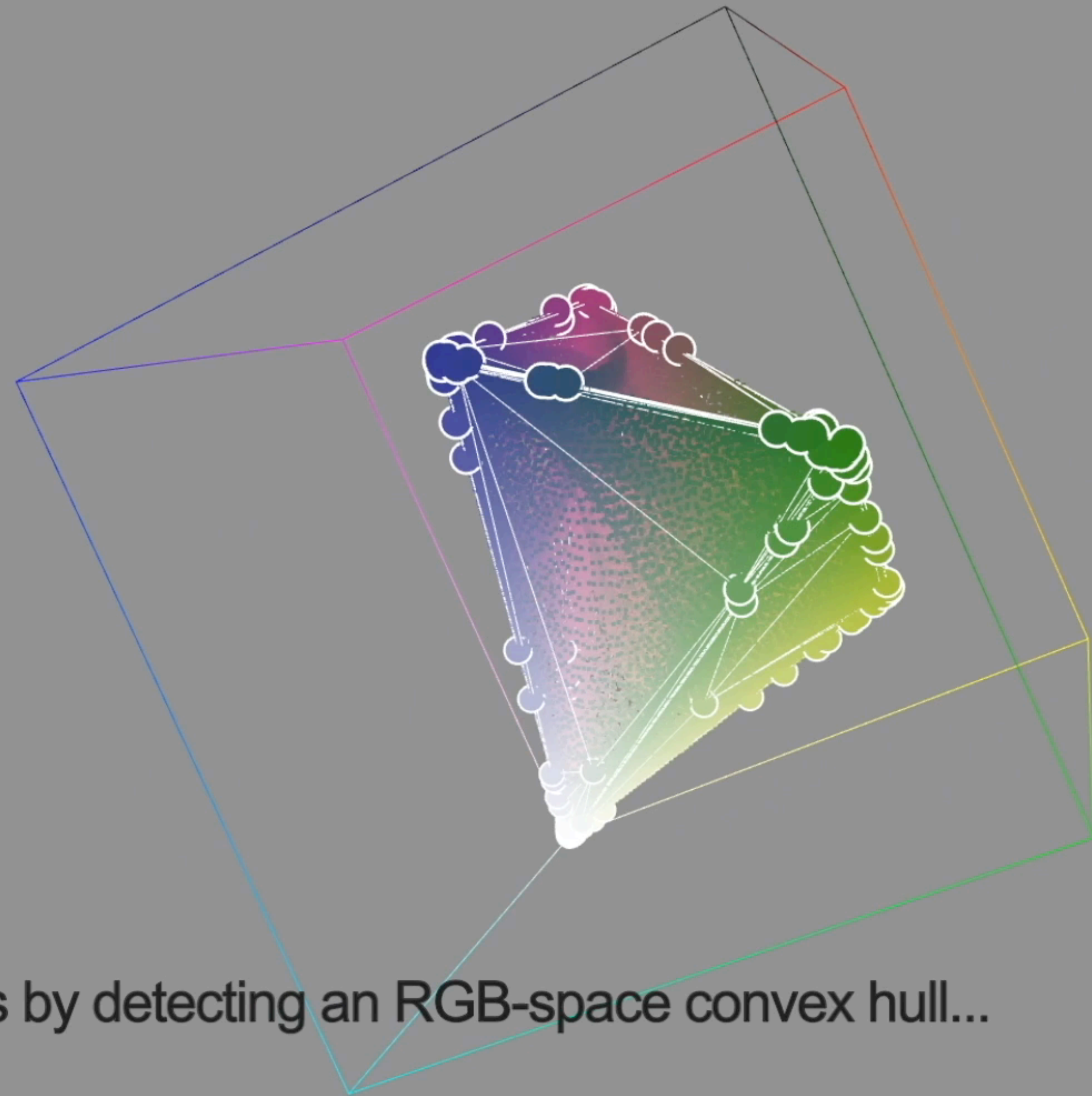




Input



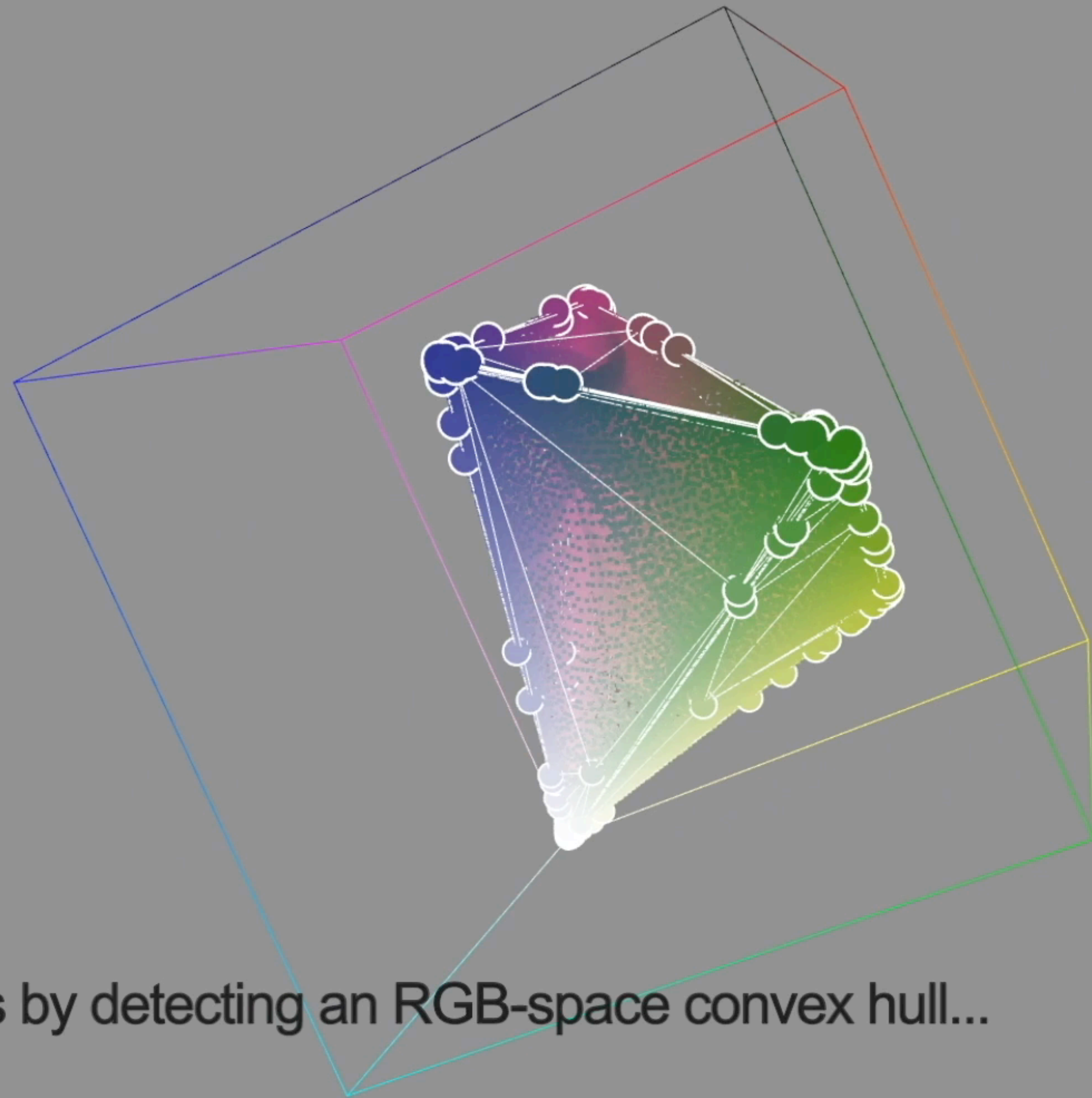
Edit



We can work backwards by detecting an RGB-space convex hull...

Palette selection



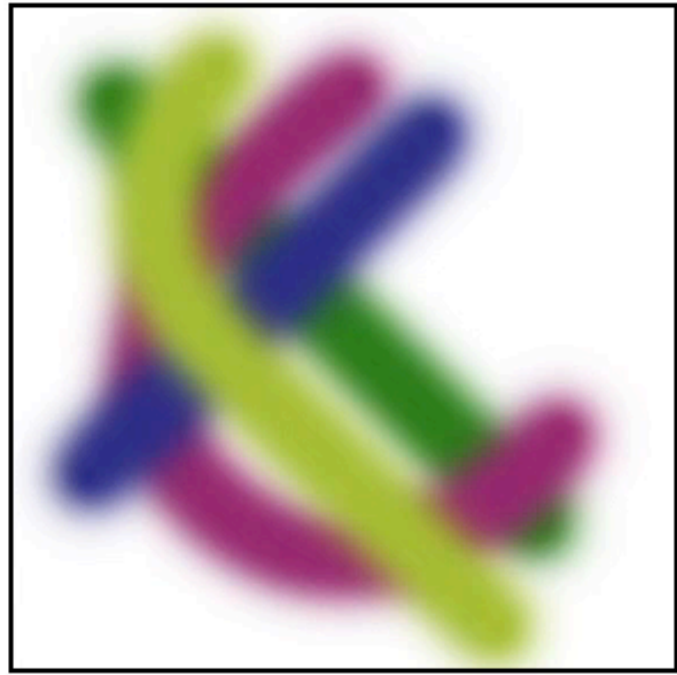


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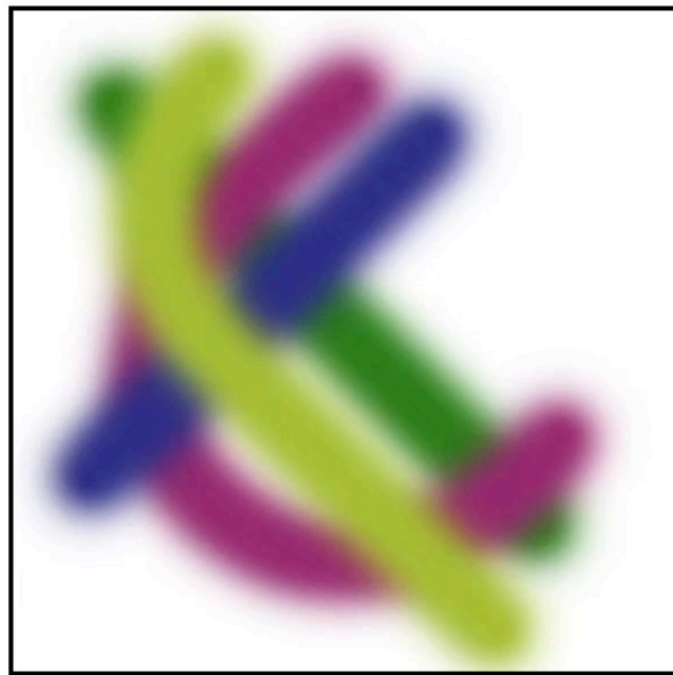
## Original



We then solve an optimization problem to extract translucent layers.

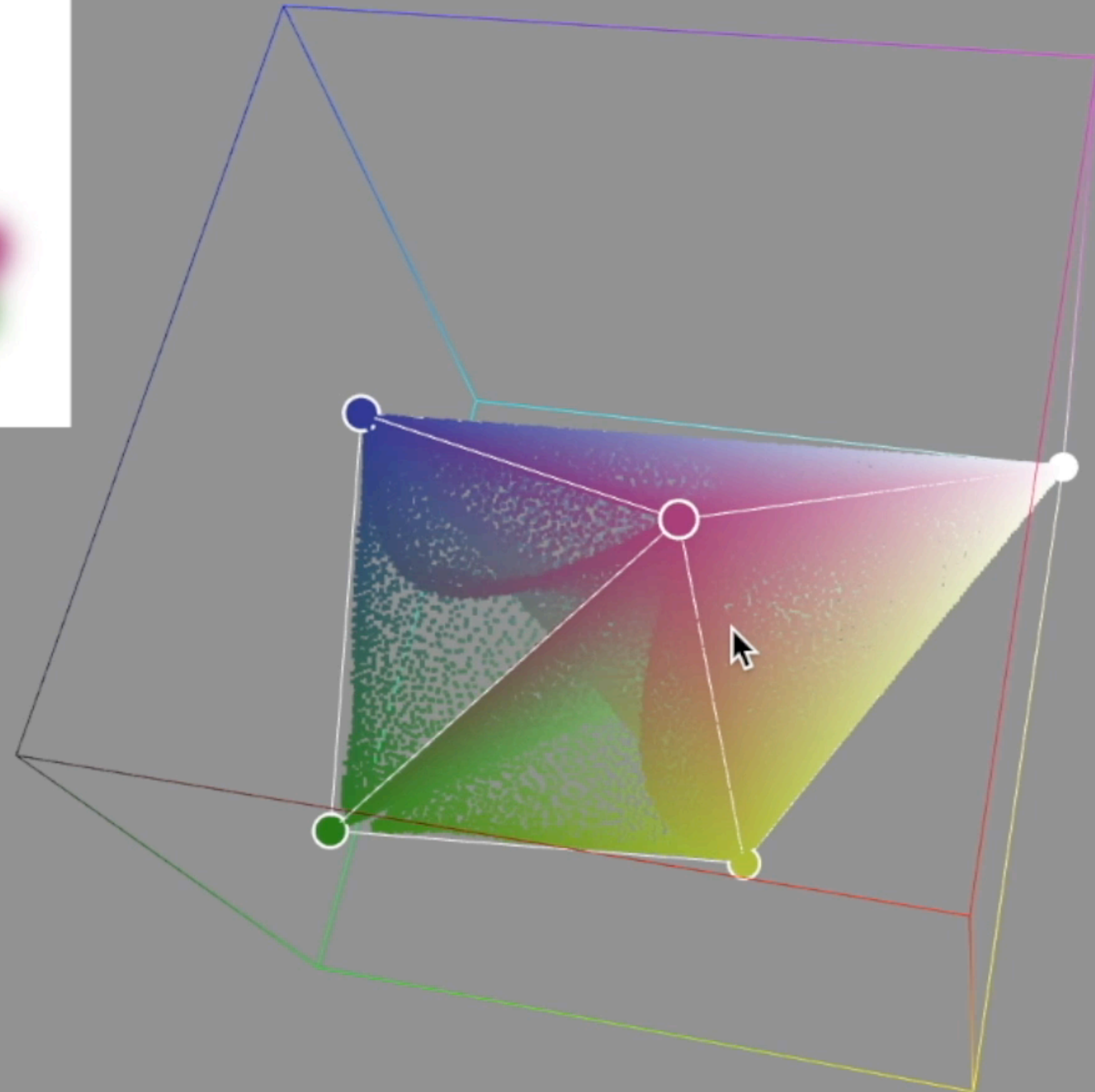
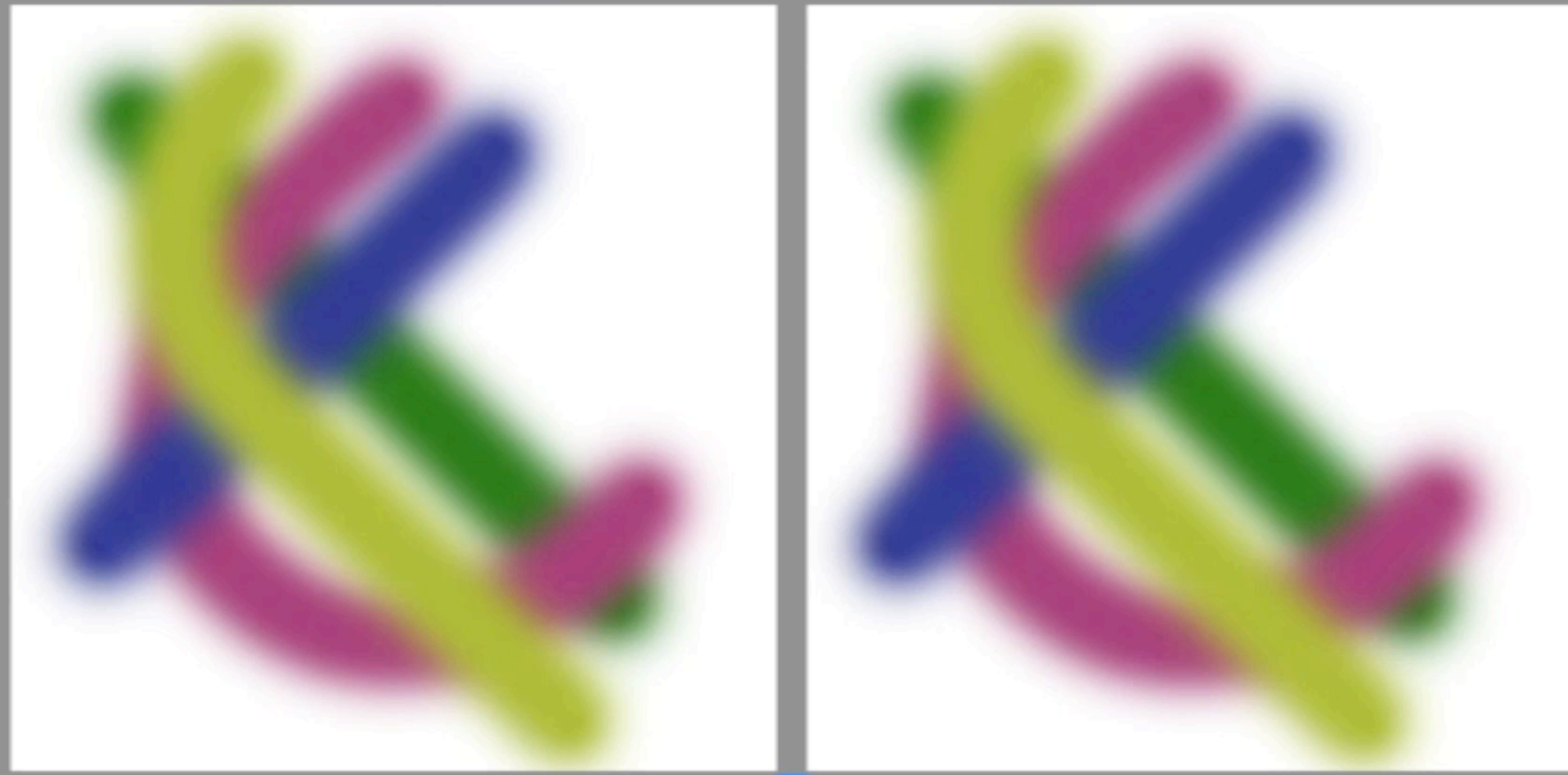
Layer opacity

**Original**



We then solve an optimization problem to extract translucent layers.

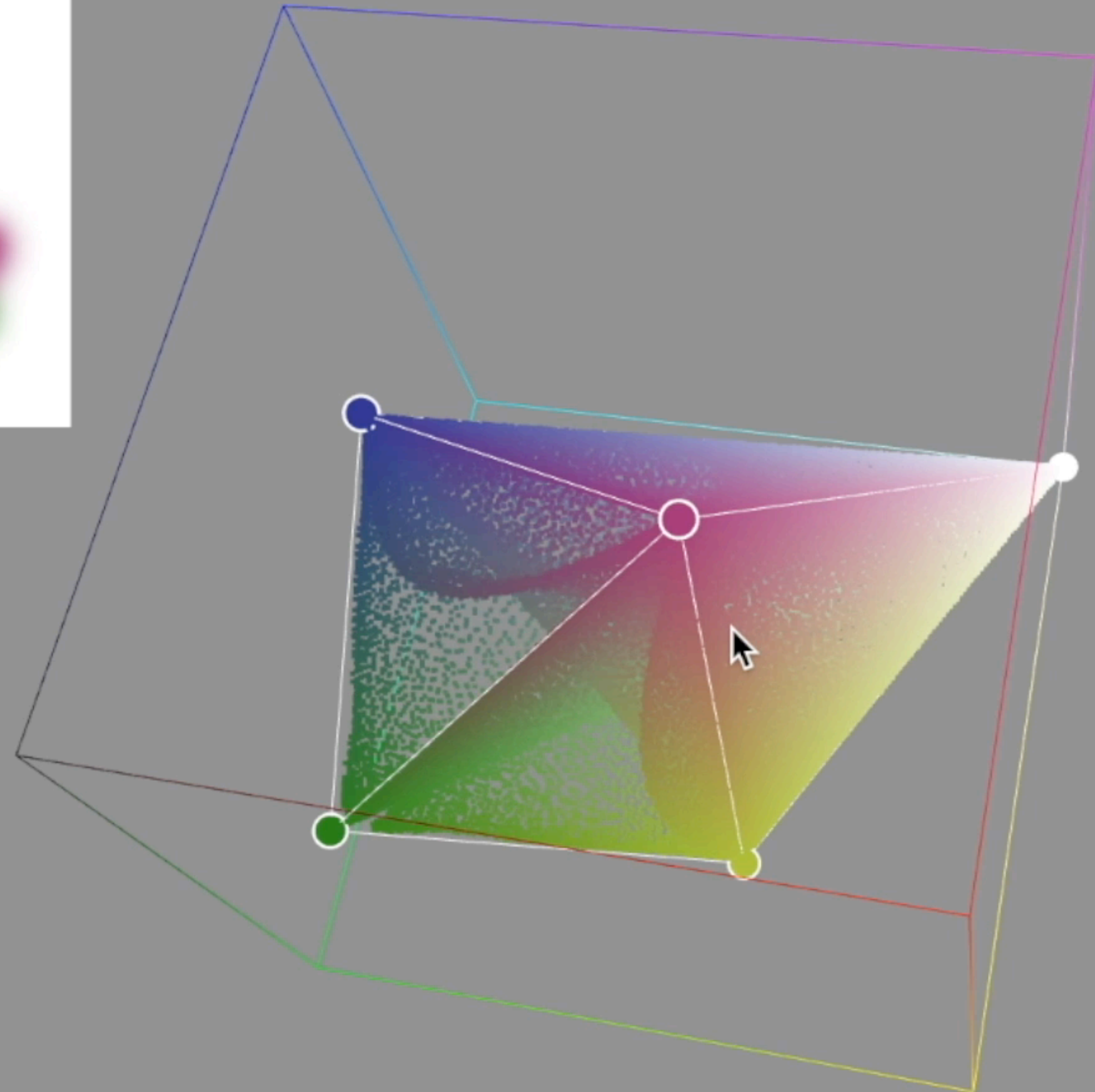
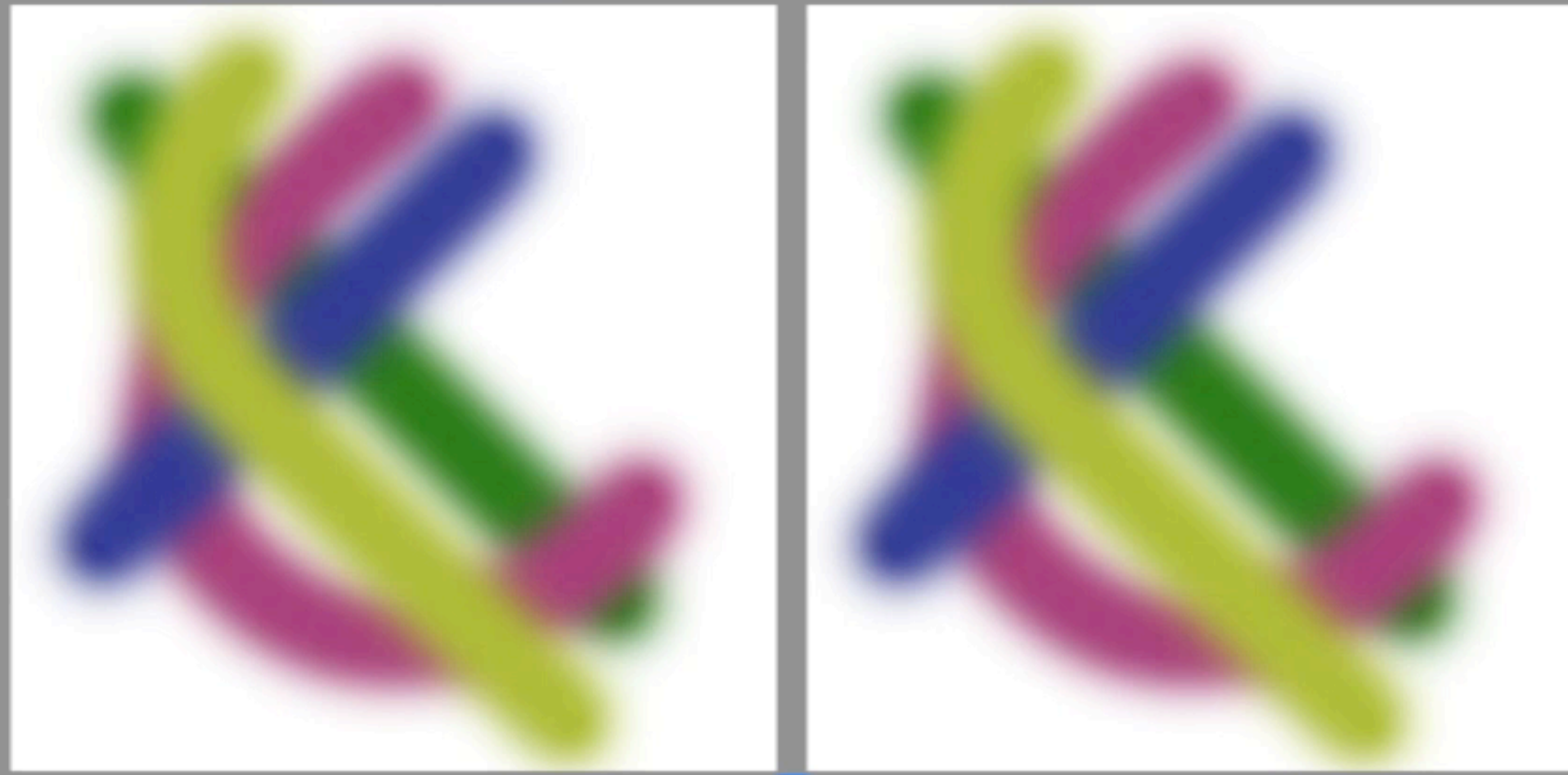
Layer opacity



The layer opacities can be converted to barycentric coordinates.

Edit



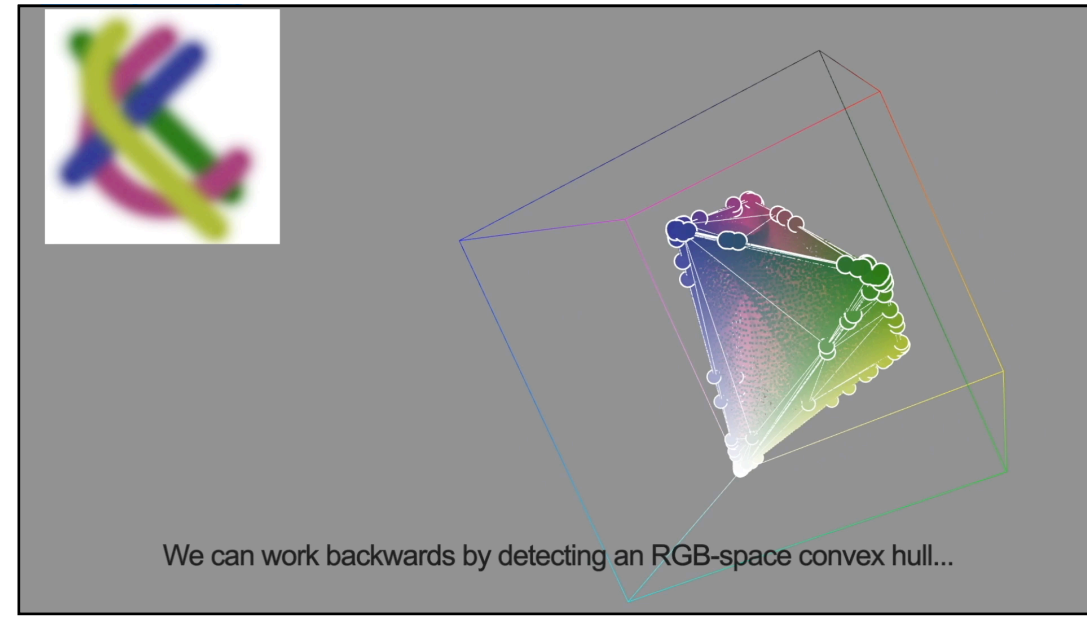


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Edit



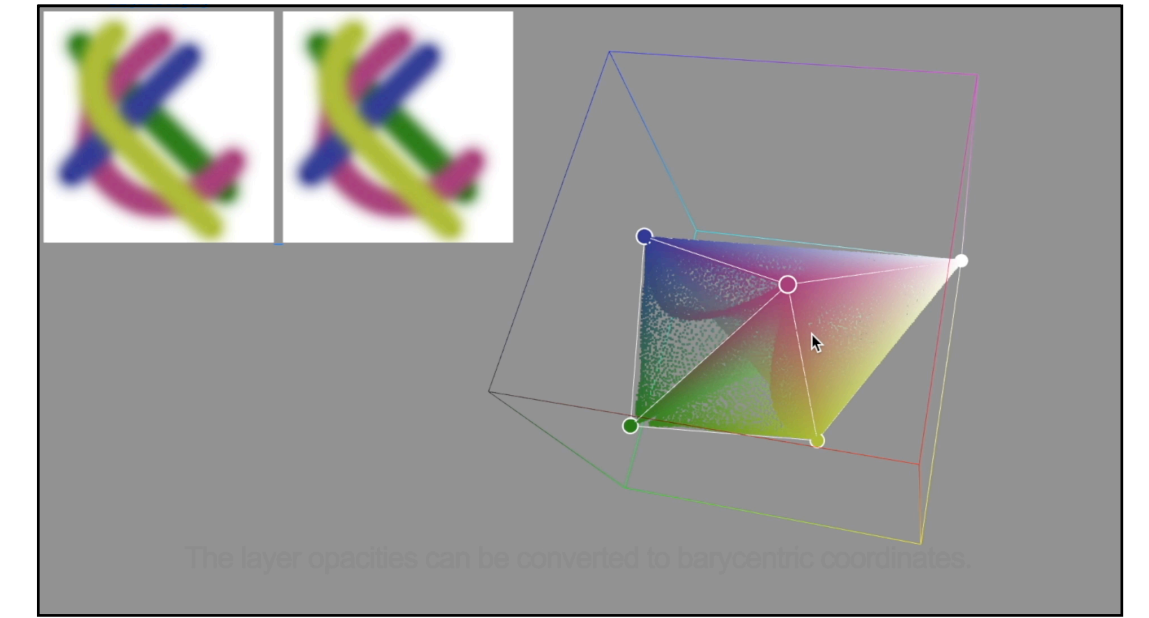
Input



Palette selection



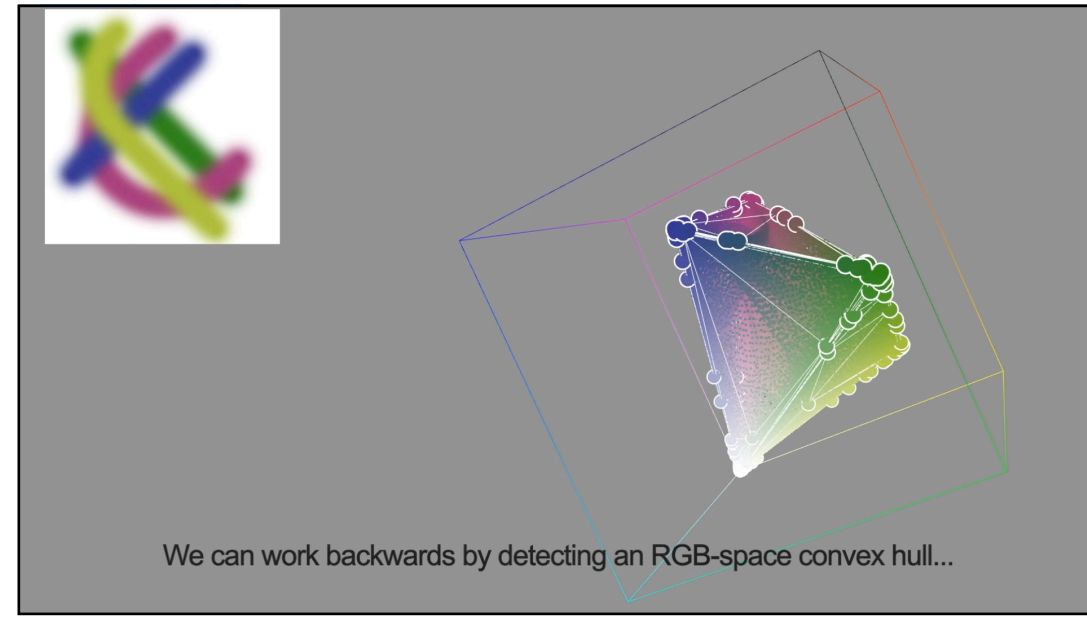
Layer opacity



Edit



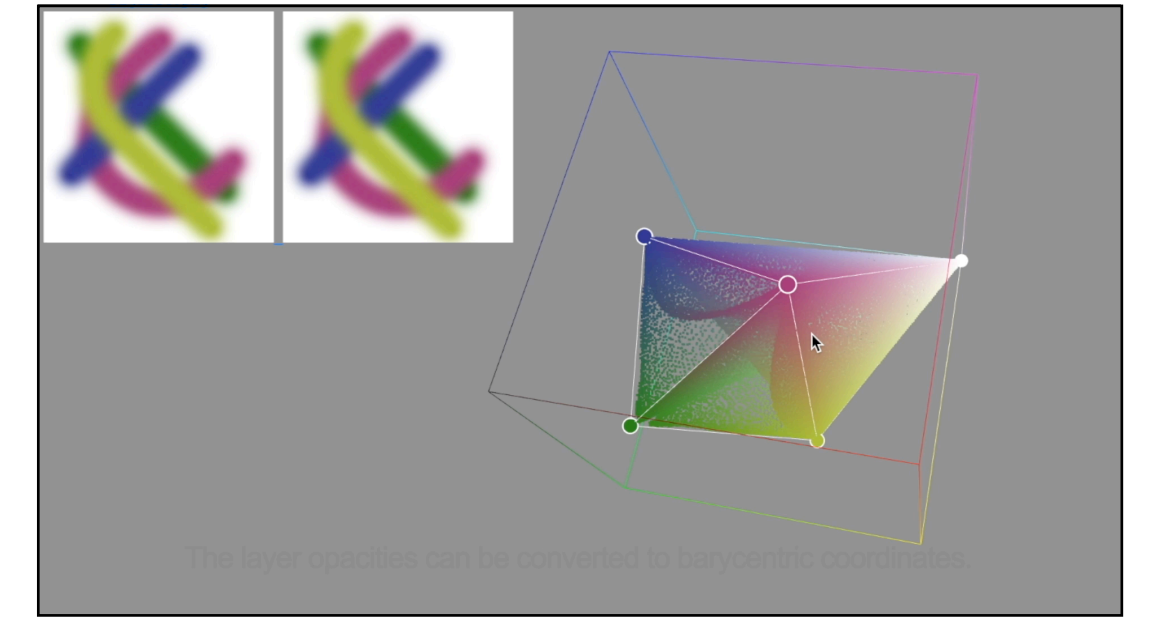
Input



Palette selection



Layer opacity



Edit



# Palette Selection

Here is an image

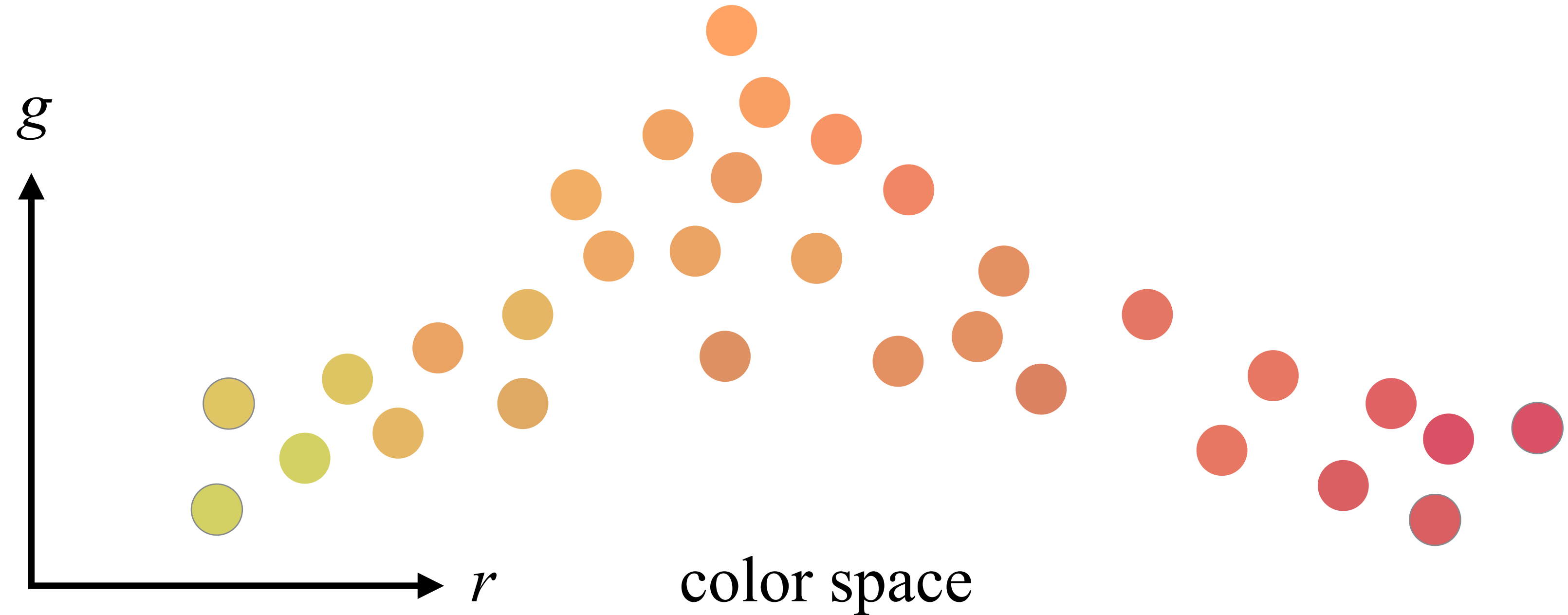


image

and its pixels in Red-Green space



image



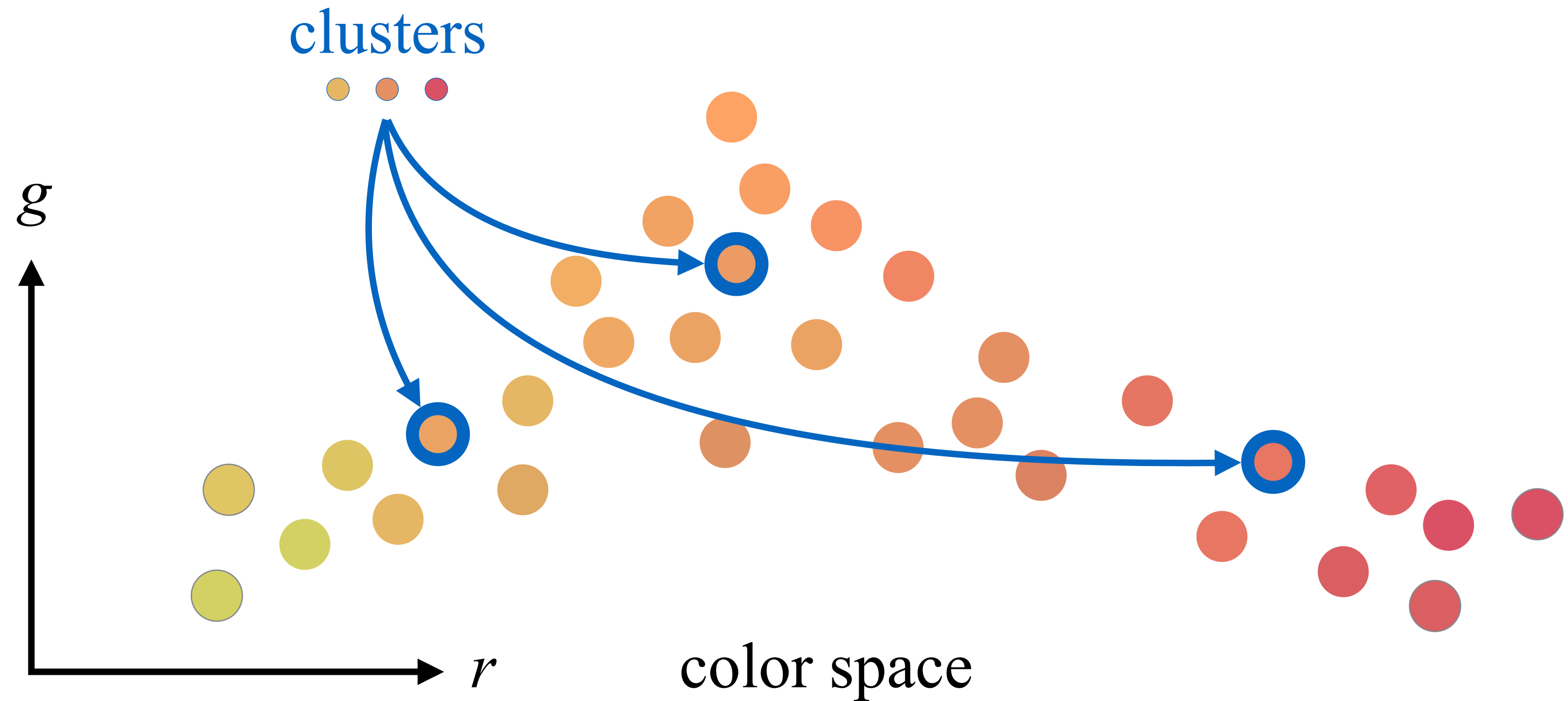
color space



# Clustering finds these interior colors



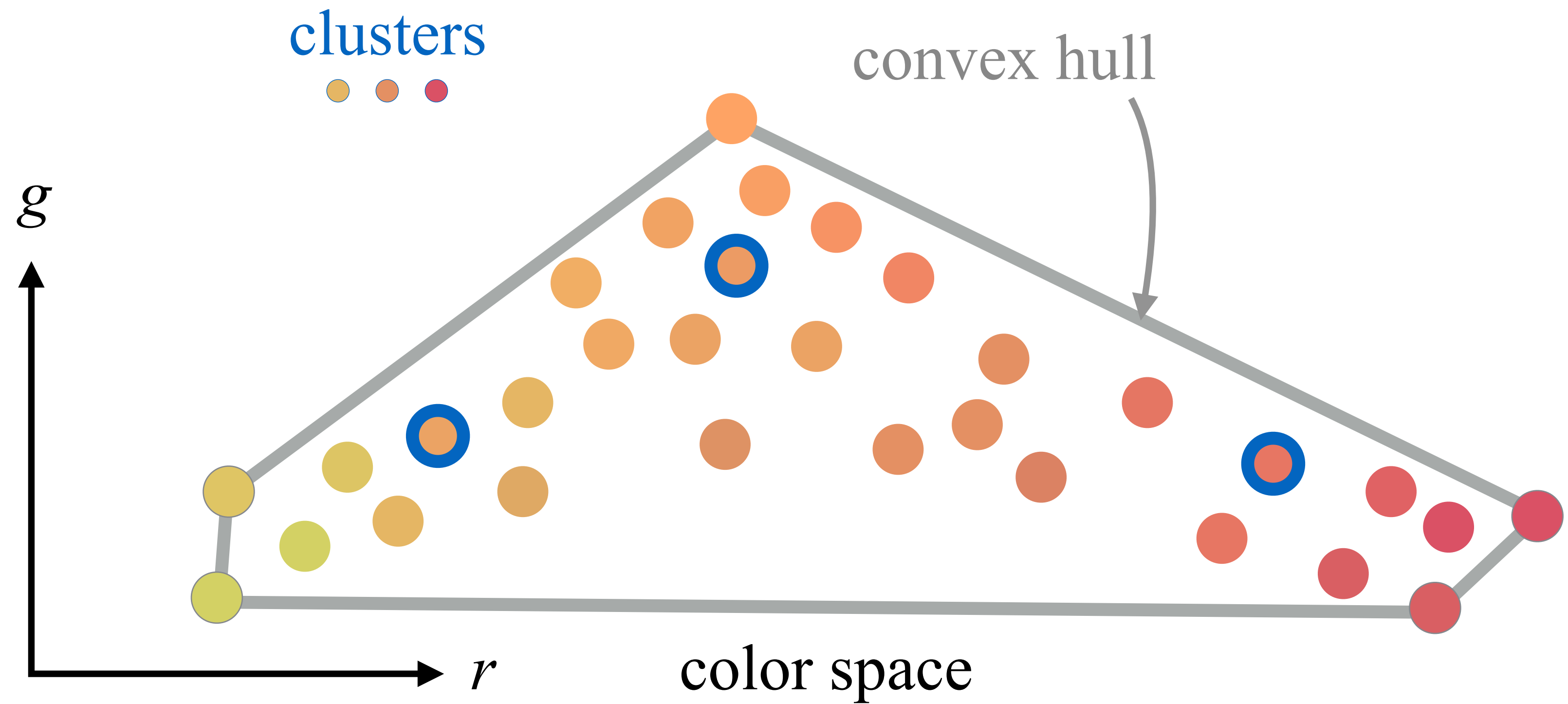
image



# The convex hull is complex



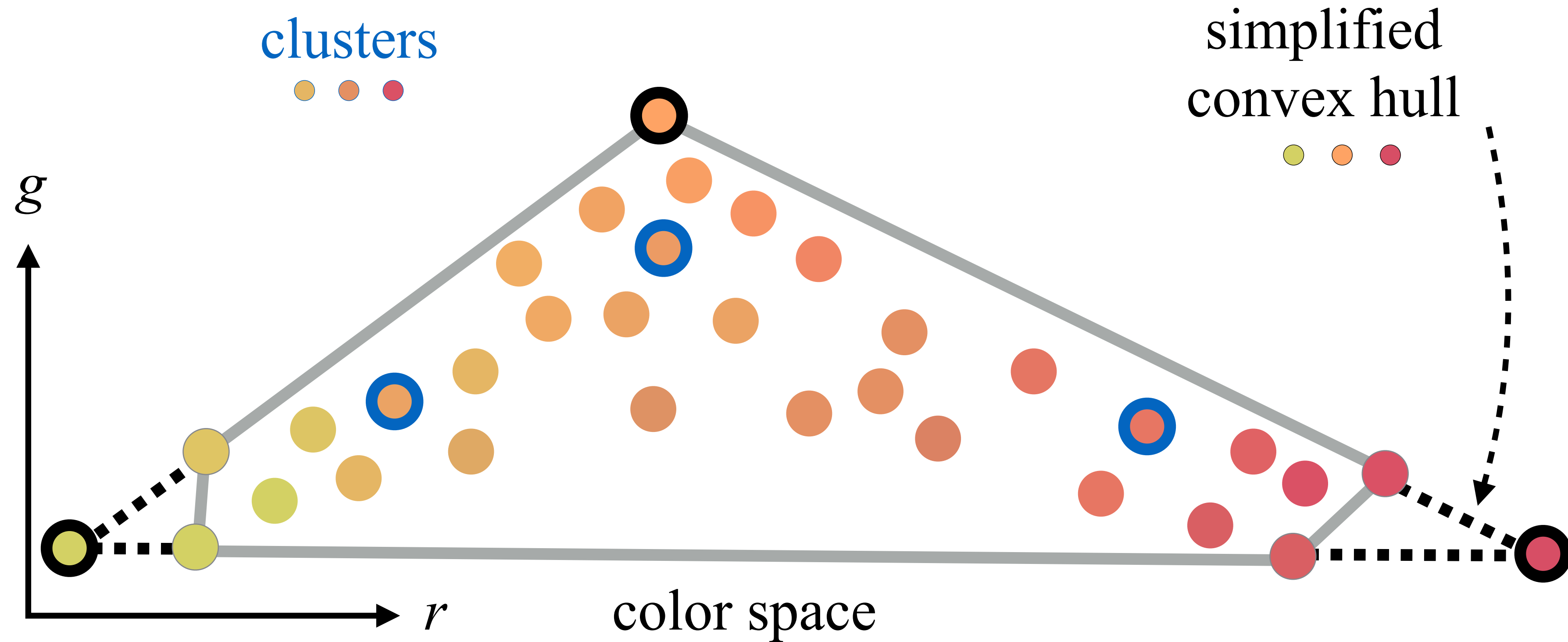
image



If we could simplify it...



image

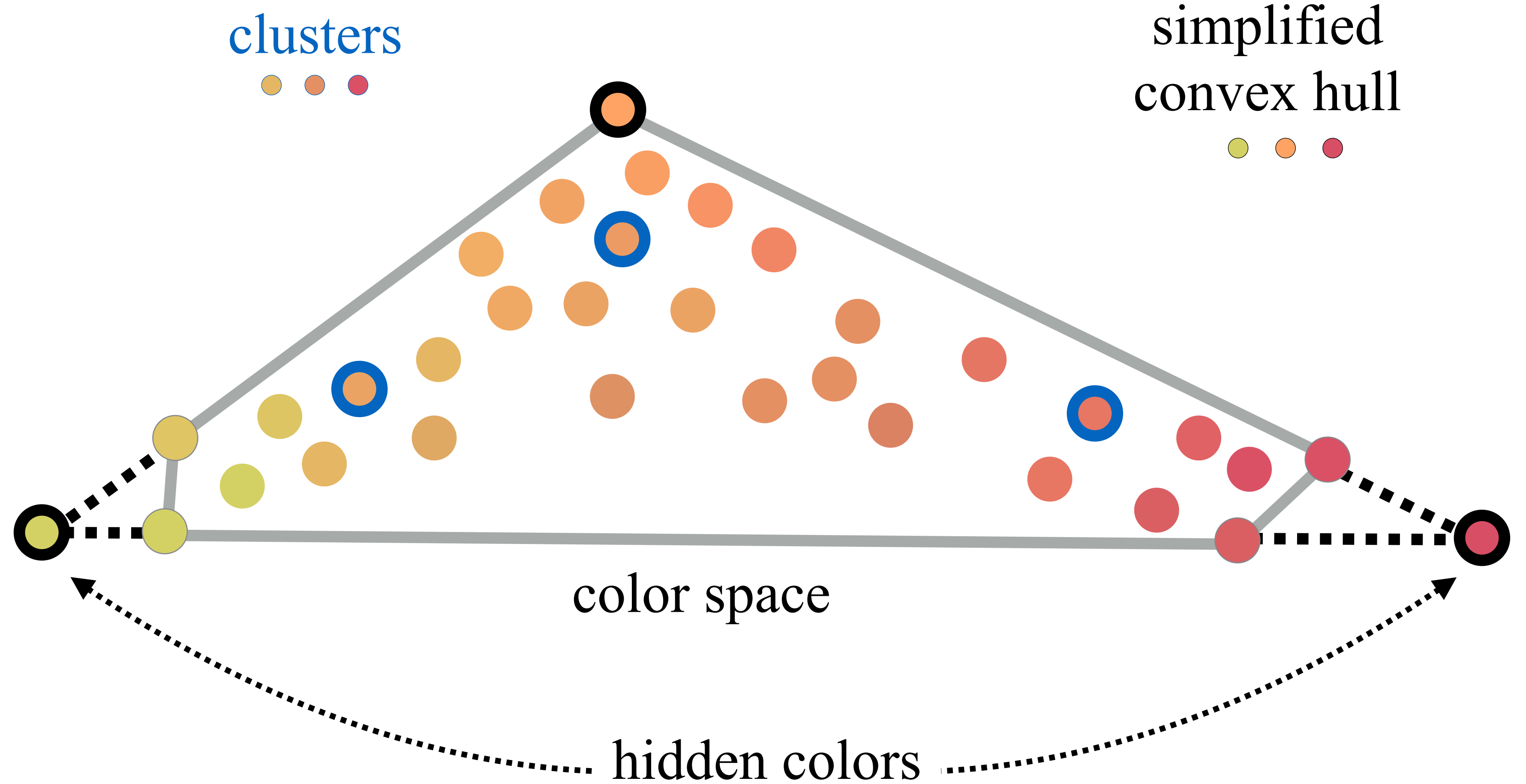




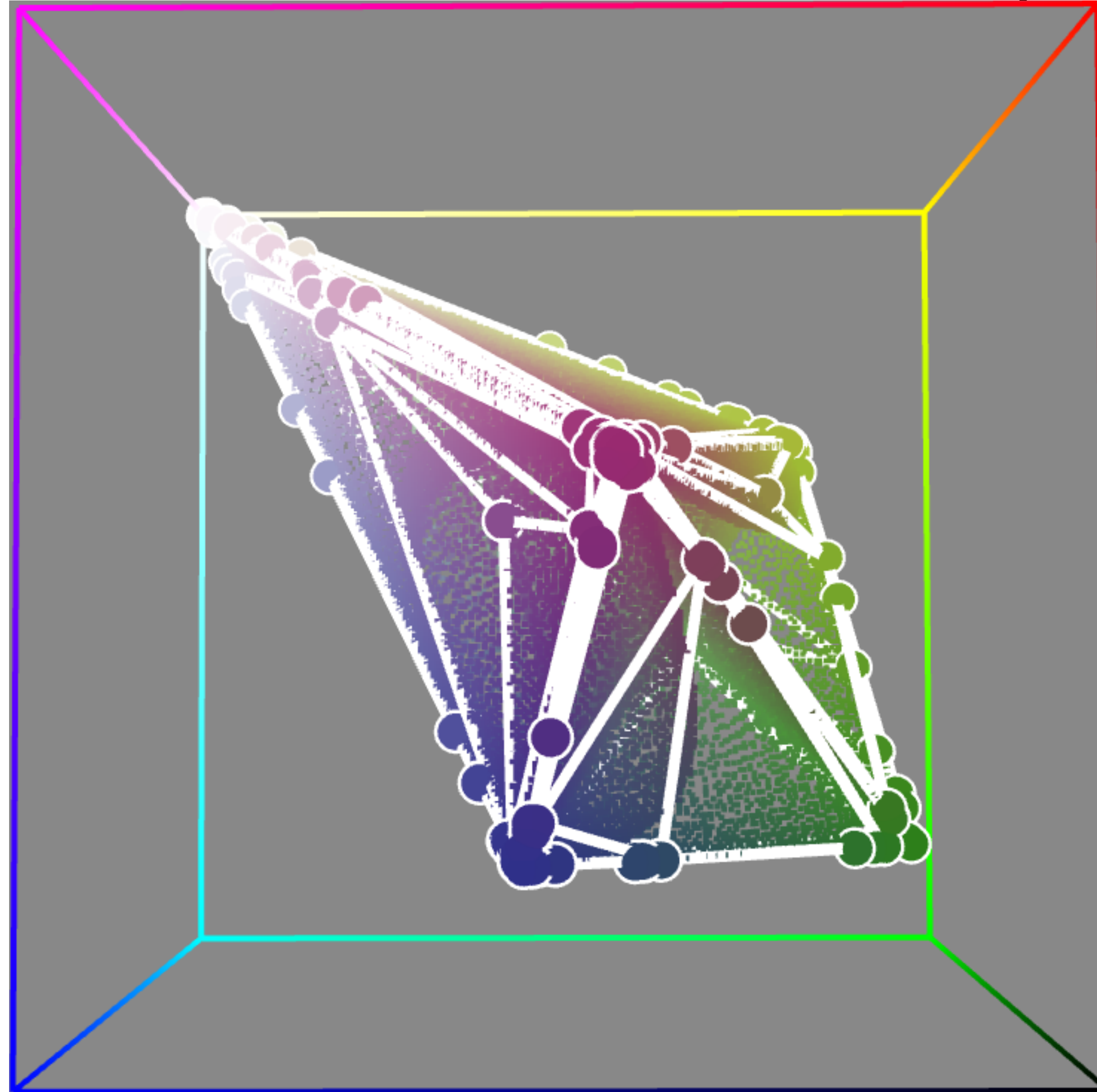
# We would find the original, hidden palette



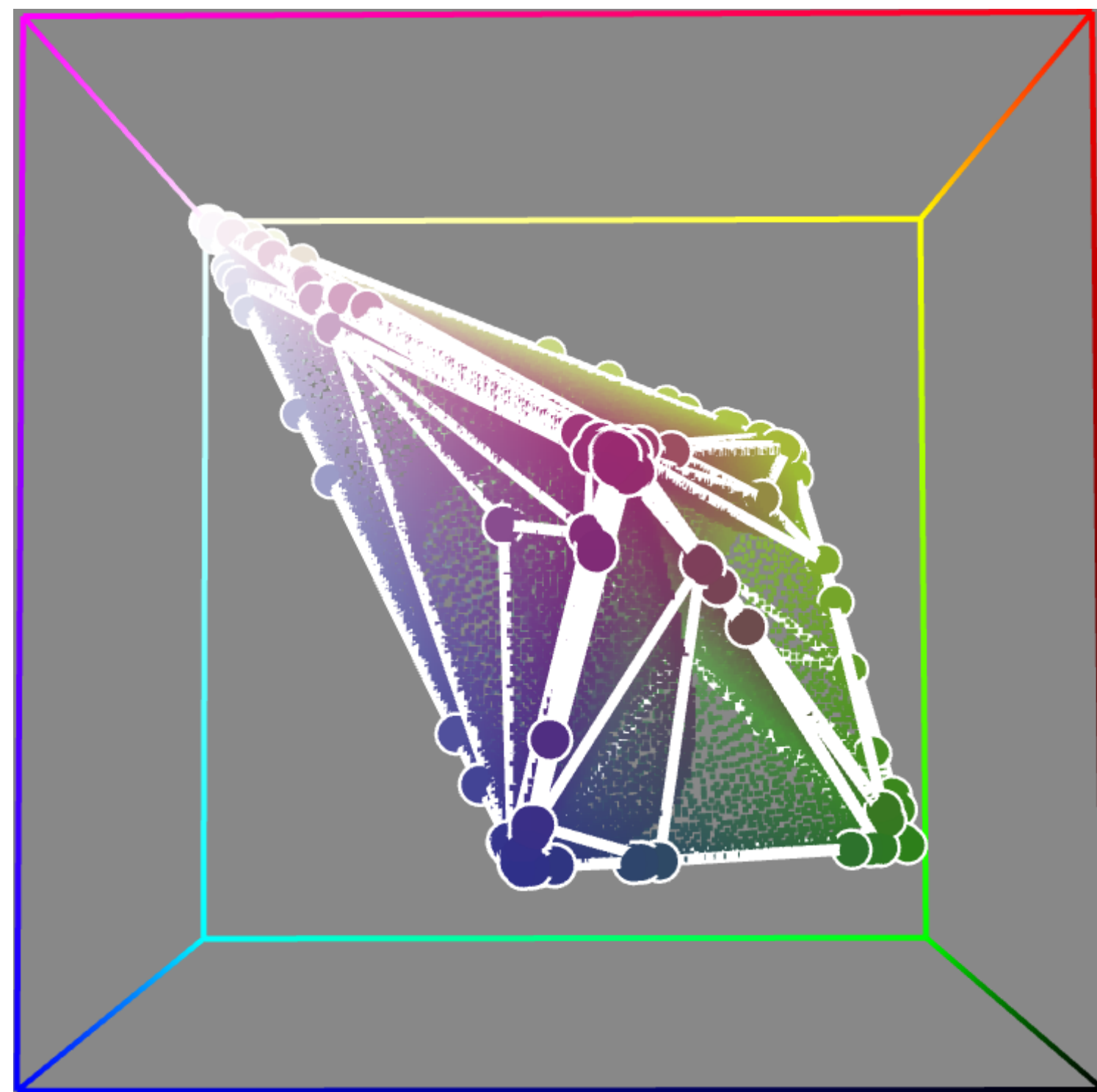
image



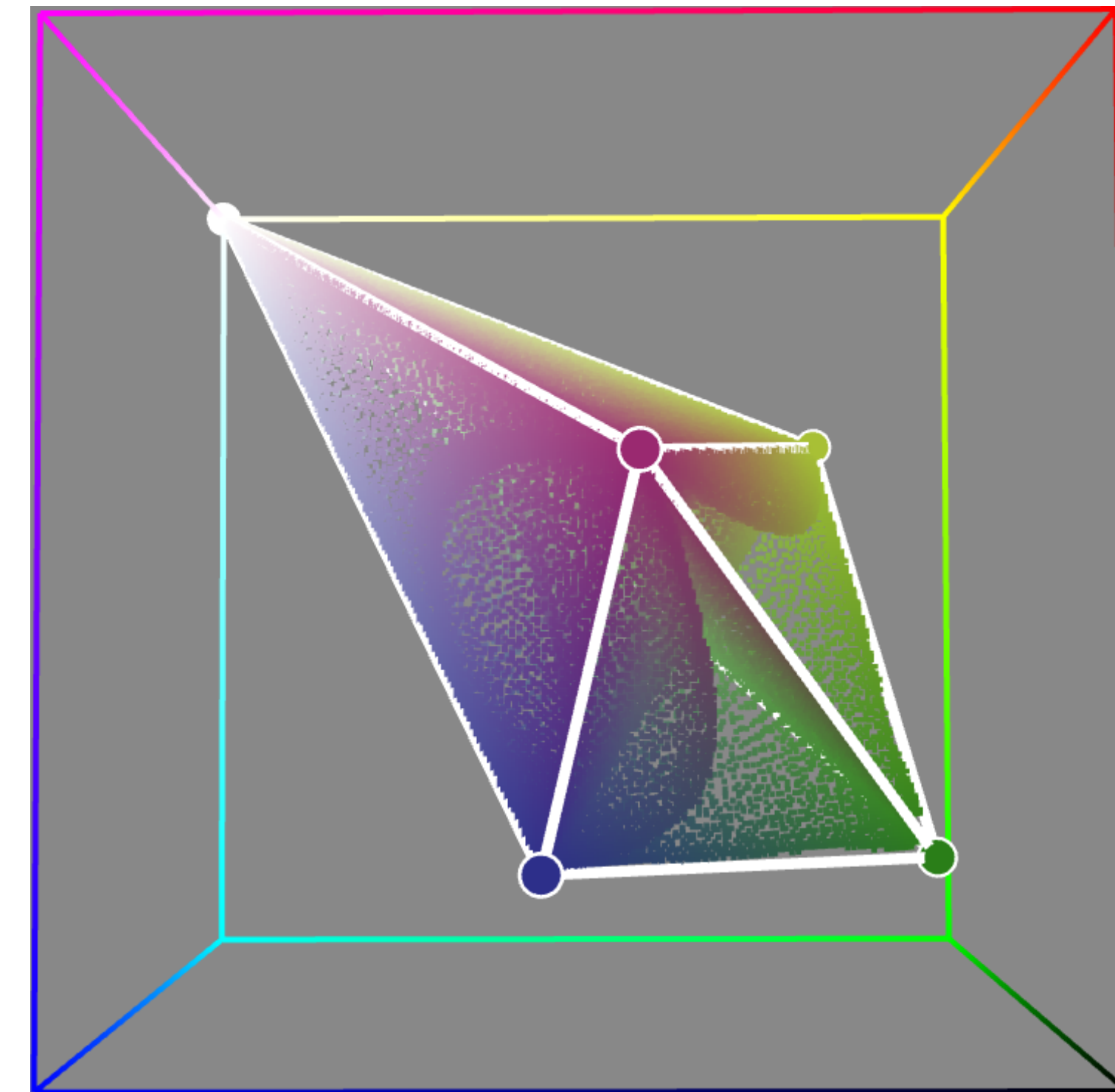
# Convex Hull in RGB-space



# Convex Hull simplification



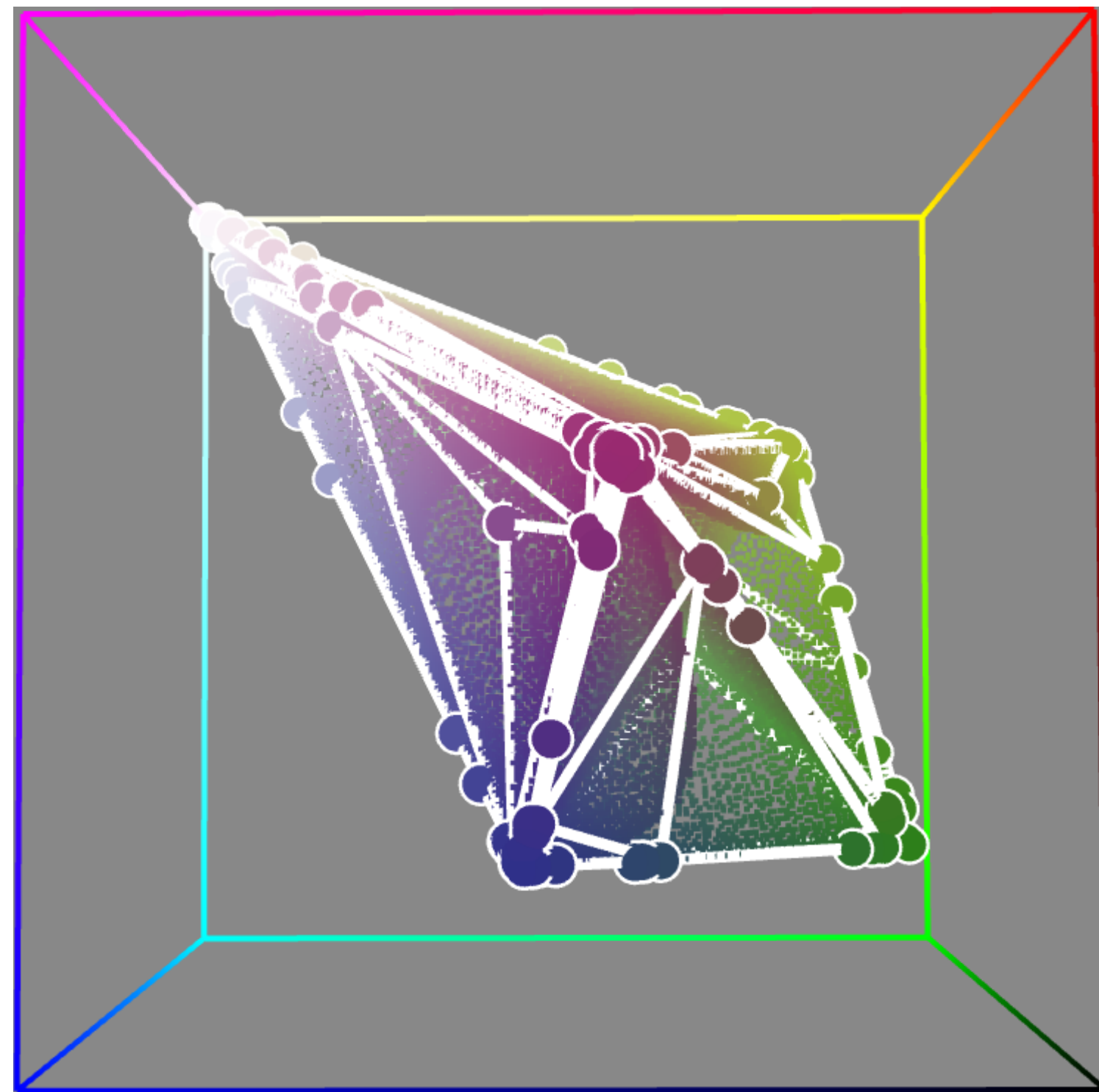
convex hull



simplified convex hull

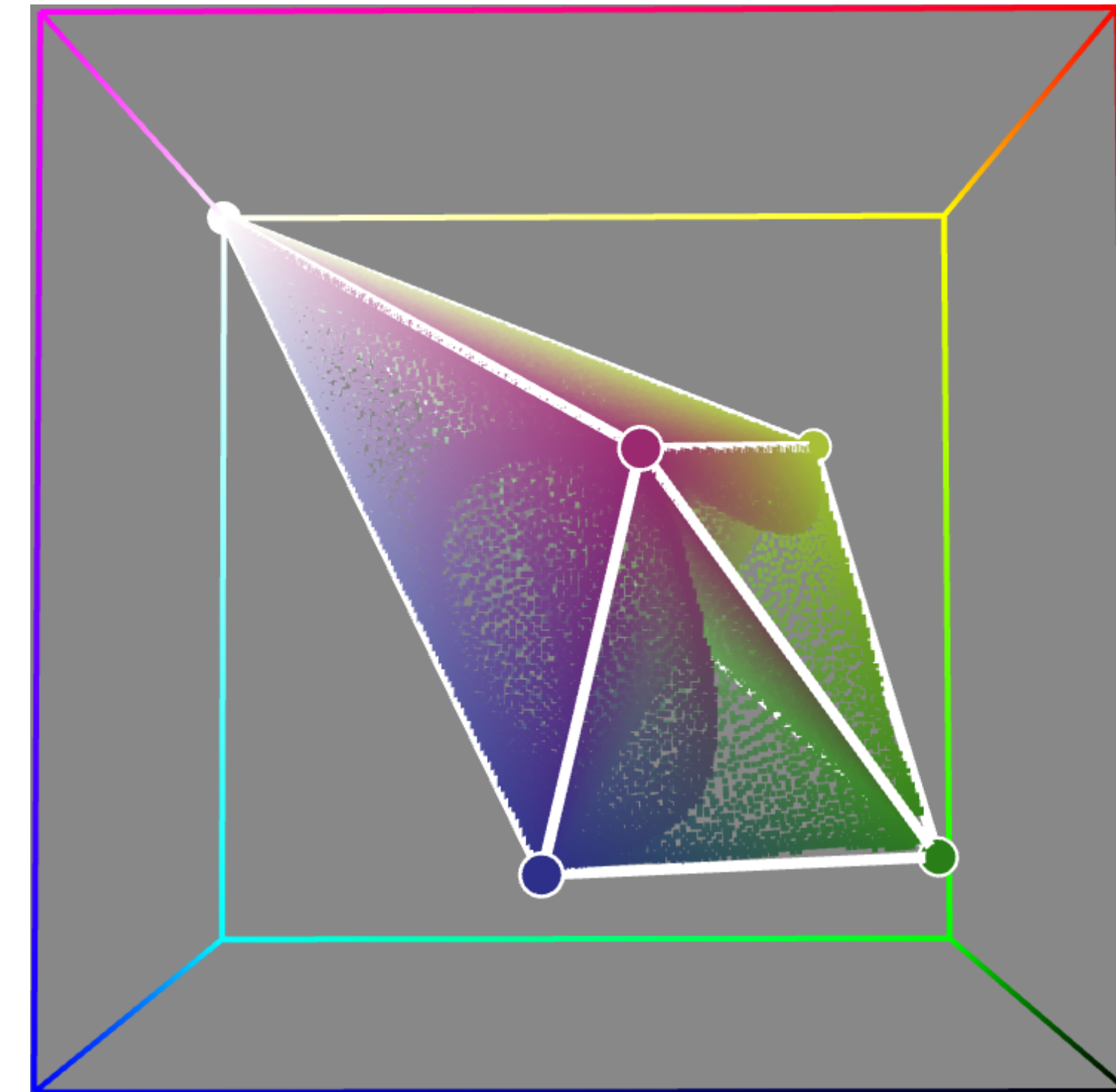


# Convex Hull simplification



convex hull

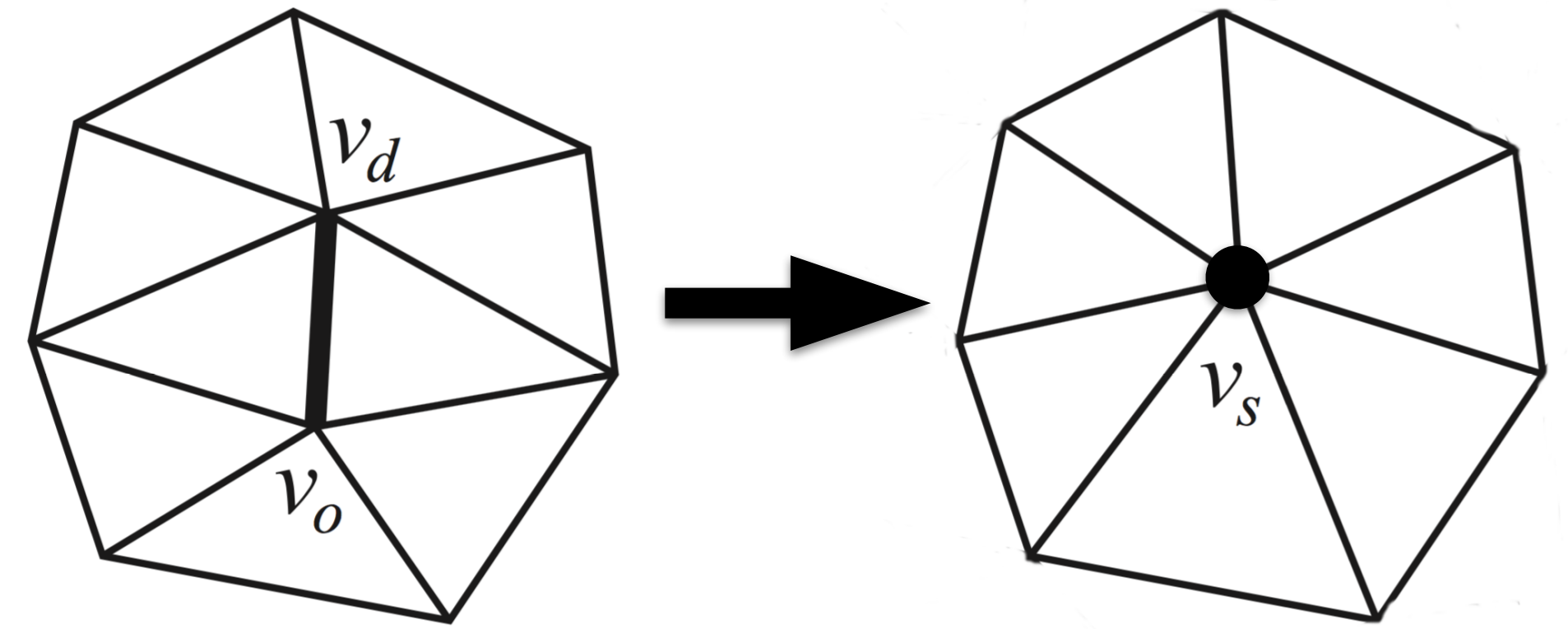
Iteratively collapse edges with a  
—————→  
modified Progressive Hull method



simplified convex hull

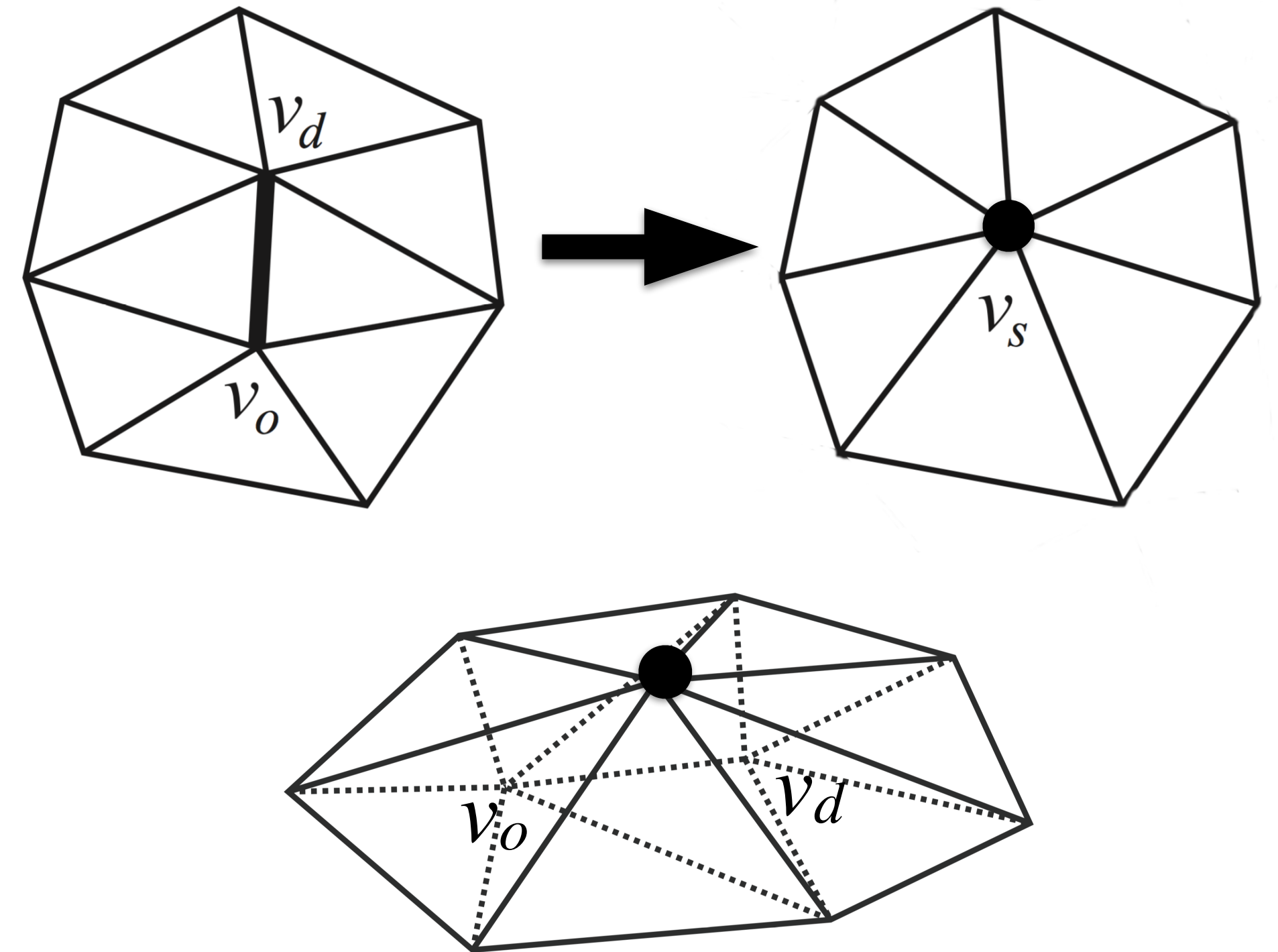
# Progressive Hull [Sander et al. 2000]

- Greedily collapse edges whose new vertex position adds the smallest additional volume.



# Progressive Hull [Sander et al. 2000]

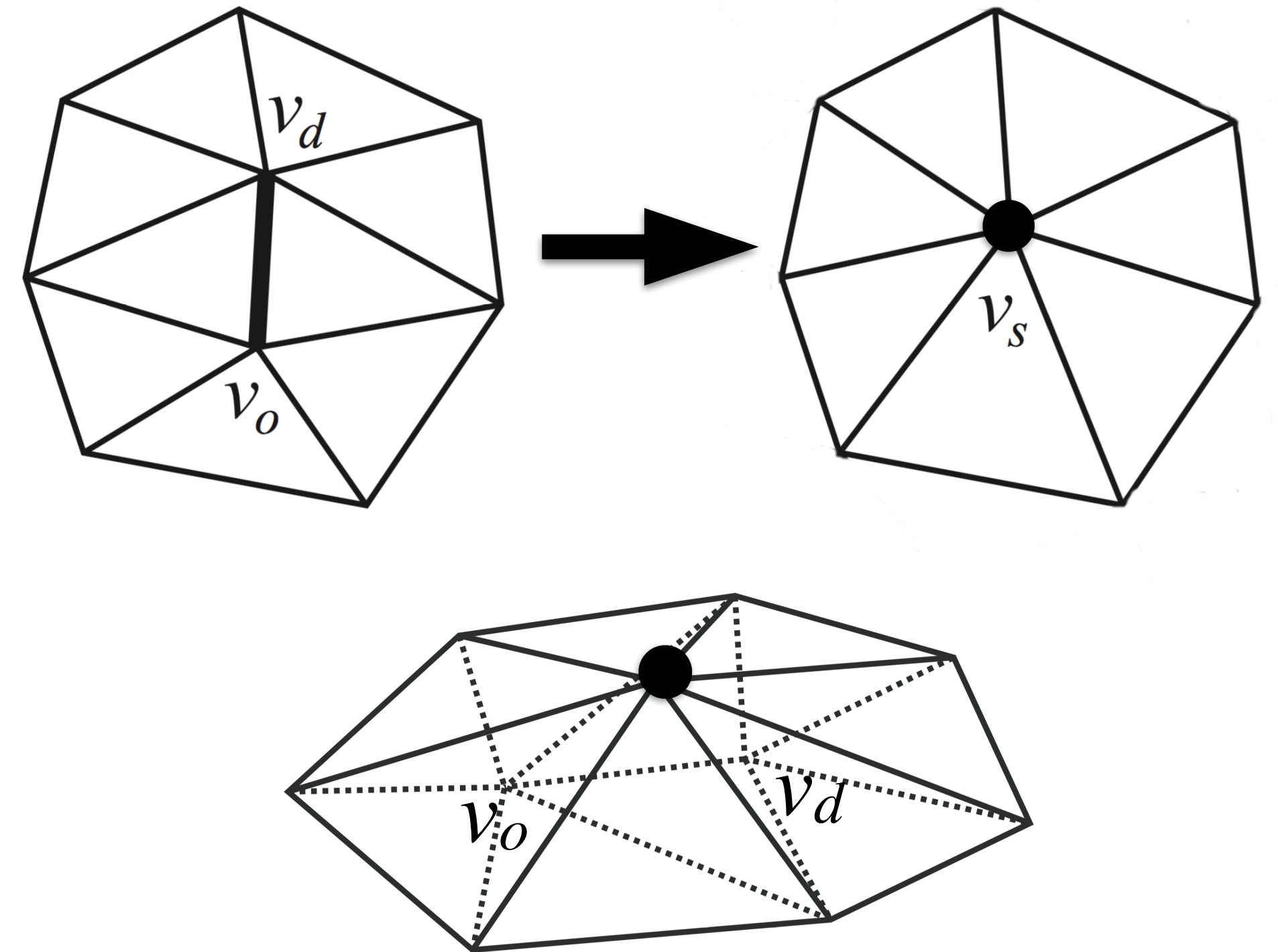
- Greedily collapse edges whose new vertex position adds the smallest additional volume.
- New vertex position guarantees that volume expands (linear constraint)





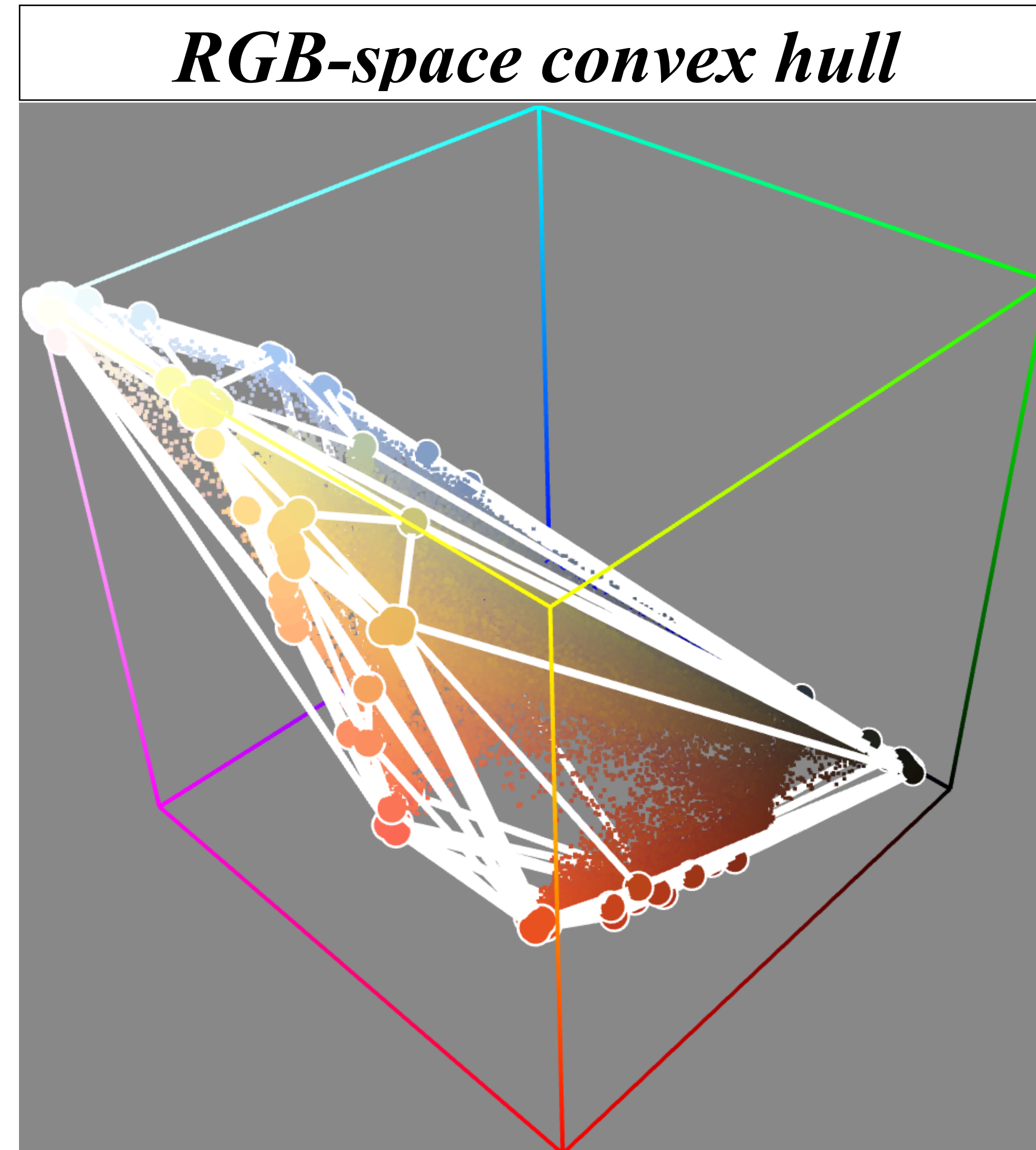
# Progressive Hull [Sander et al. 2000]

- Greedily collapse edges whose new vertex position adds the smallest additional volume.
- New vertex position guarantees that volume expands (linear constraint)
- We modify the algorithm: choose the new vertex that minimizes the distance to incident faces of the collapsing edge.



# Palette Size

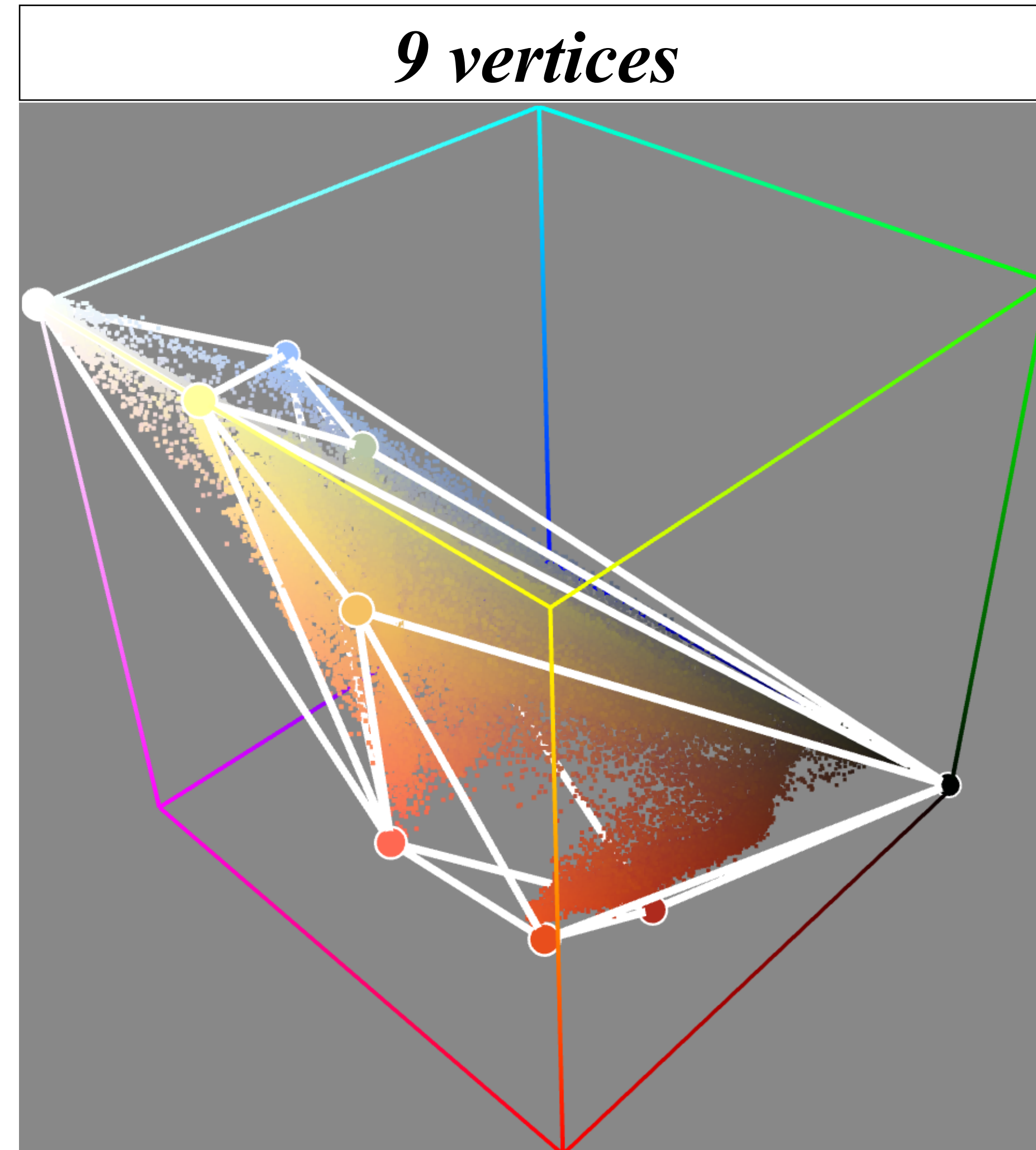
- The convex hull can be simplified to any complexity level.





# Palette Size

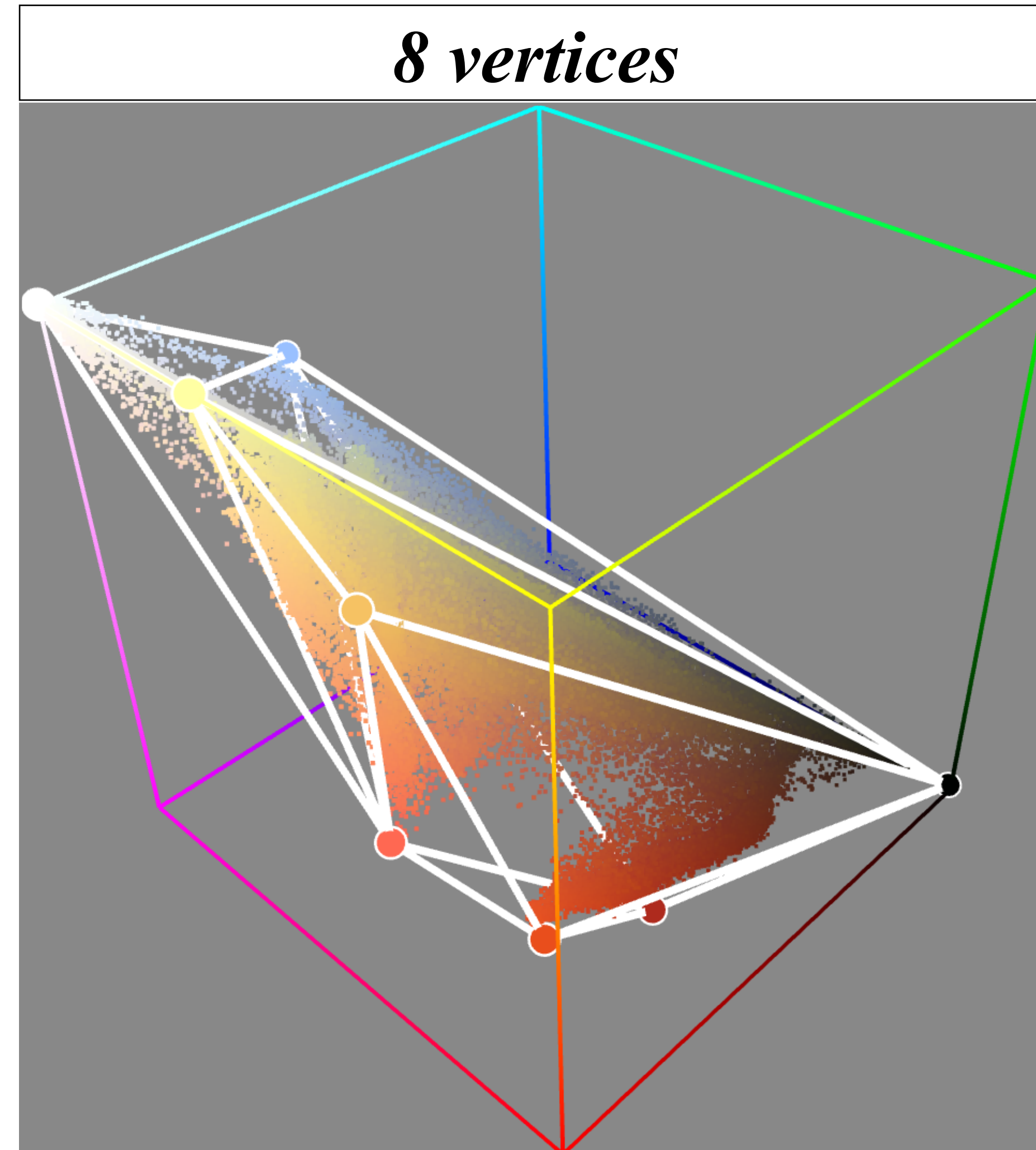
- The convex hull can be simplified to any complexity level.





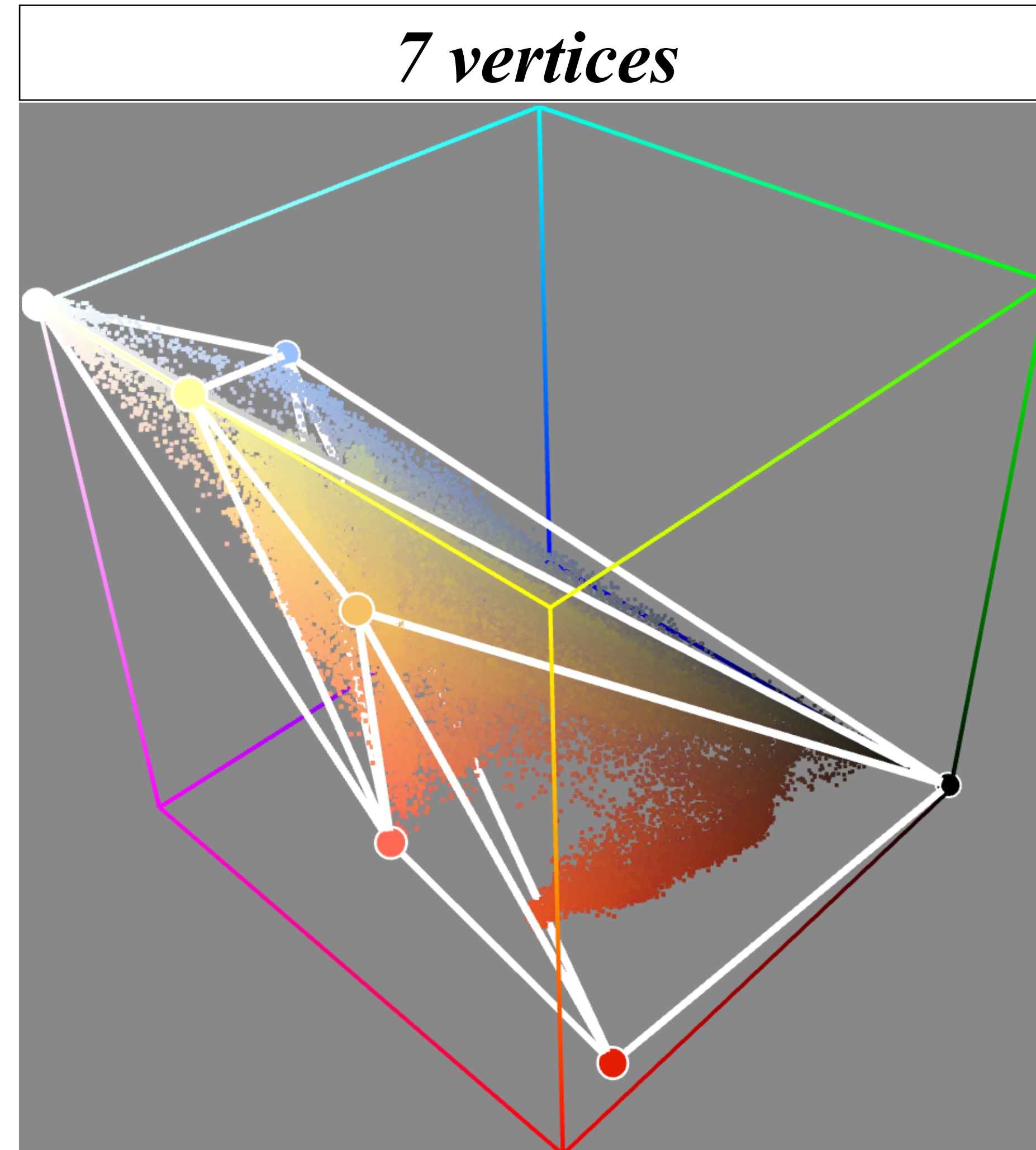
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# Palette Size

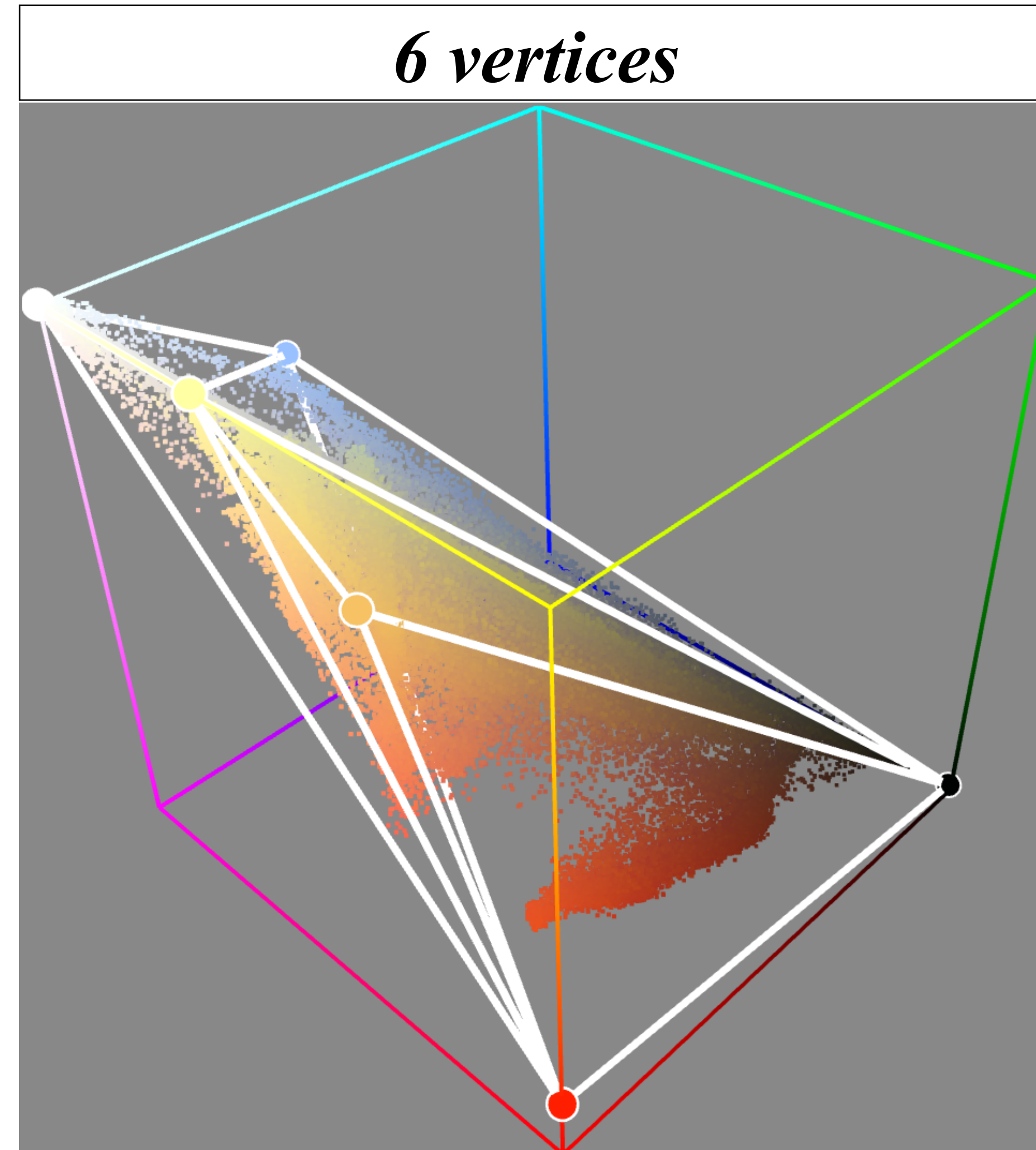
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# Palette Size

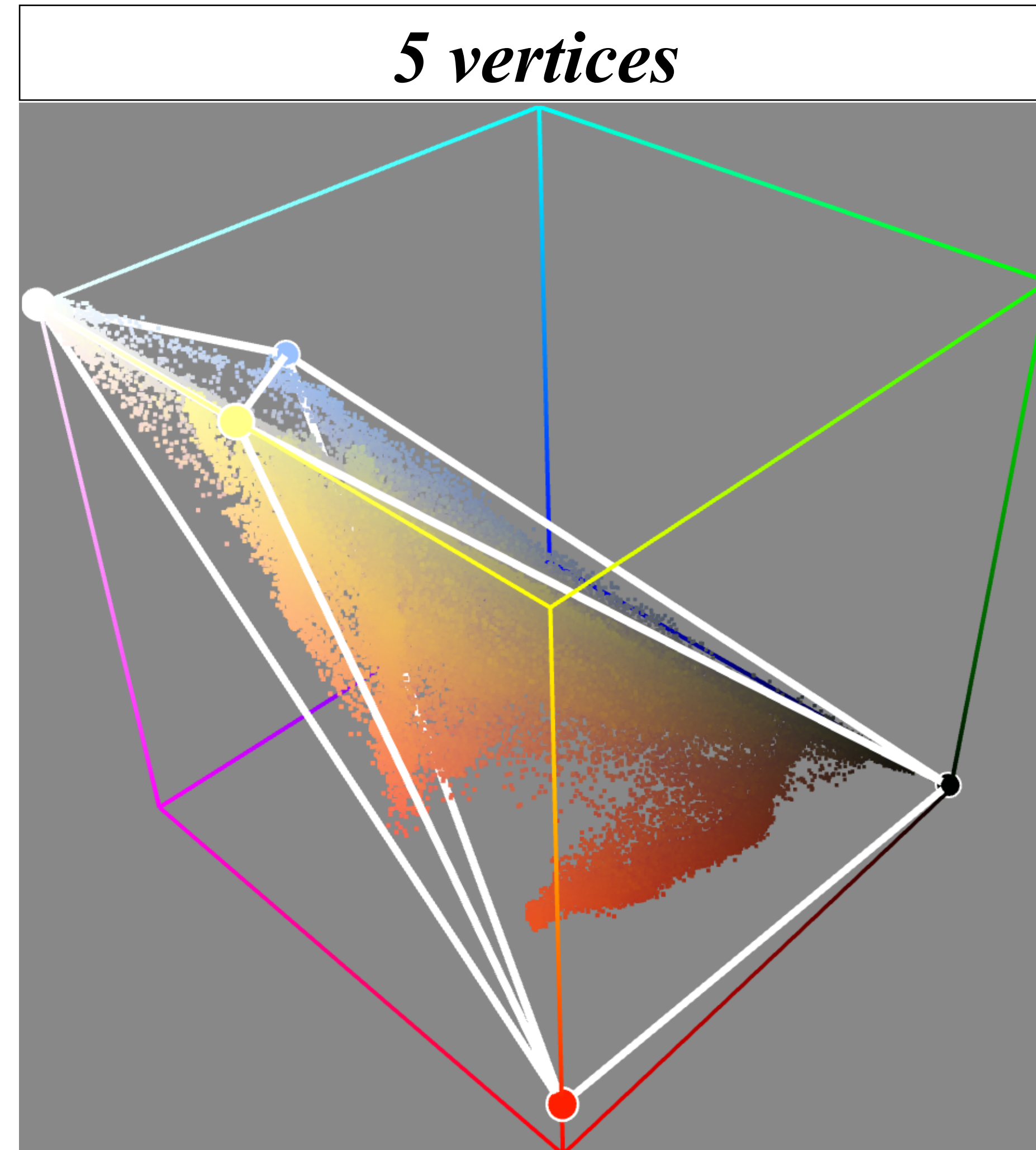
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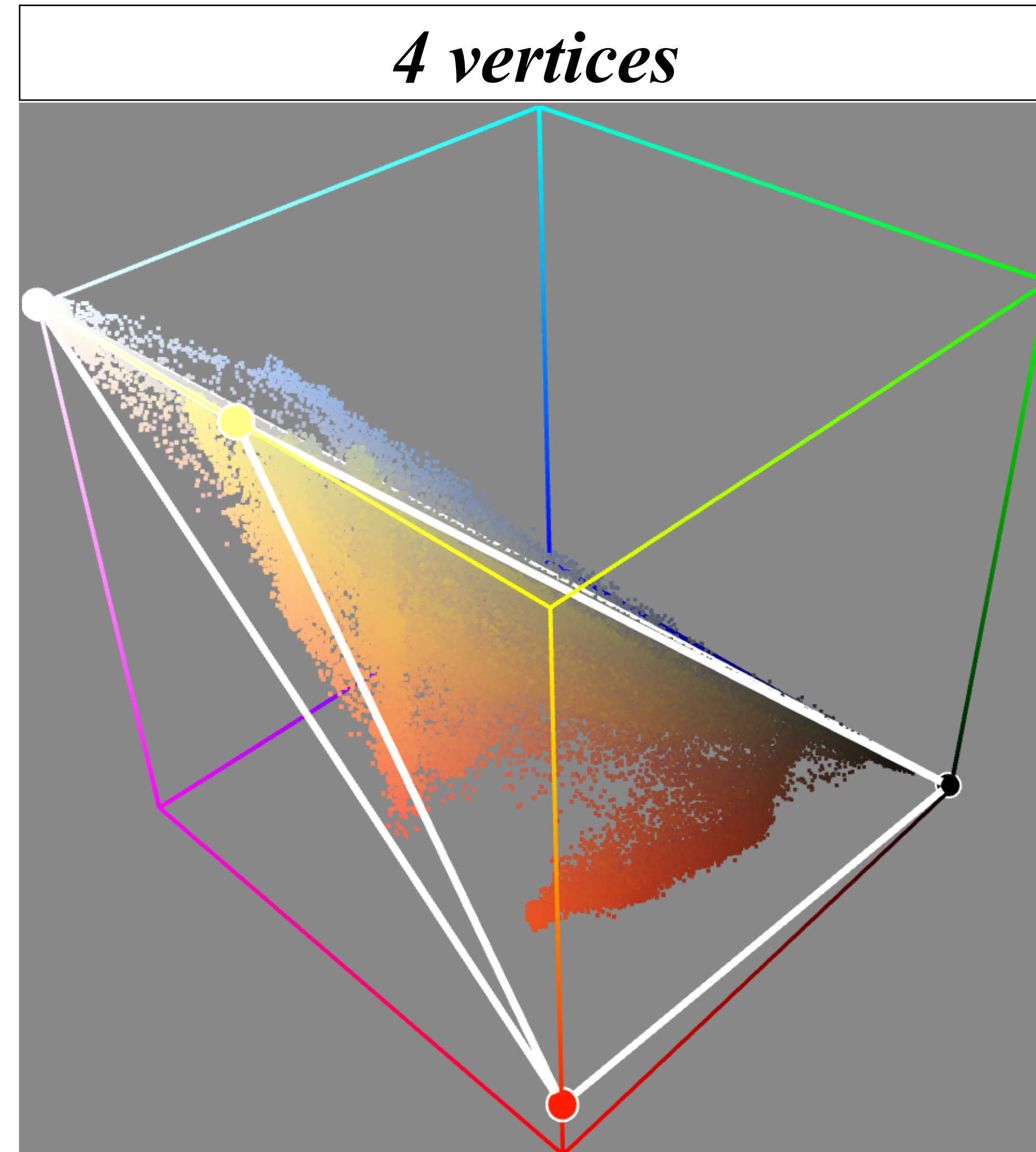
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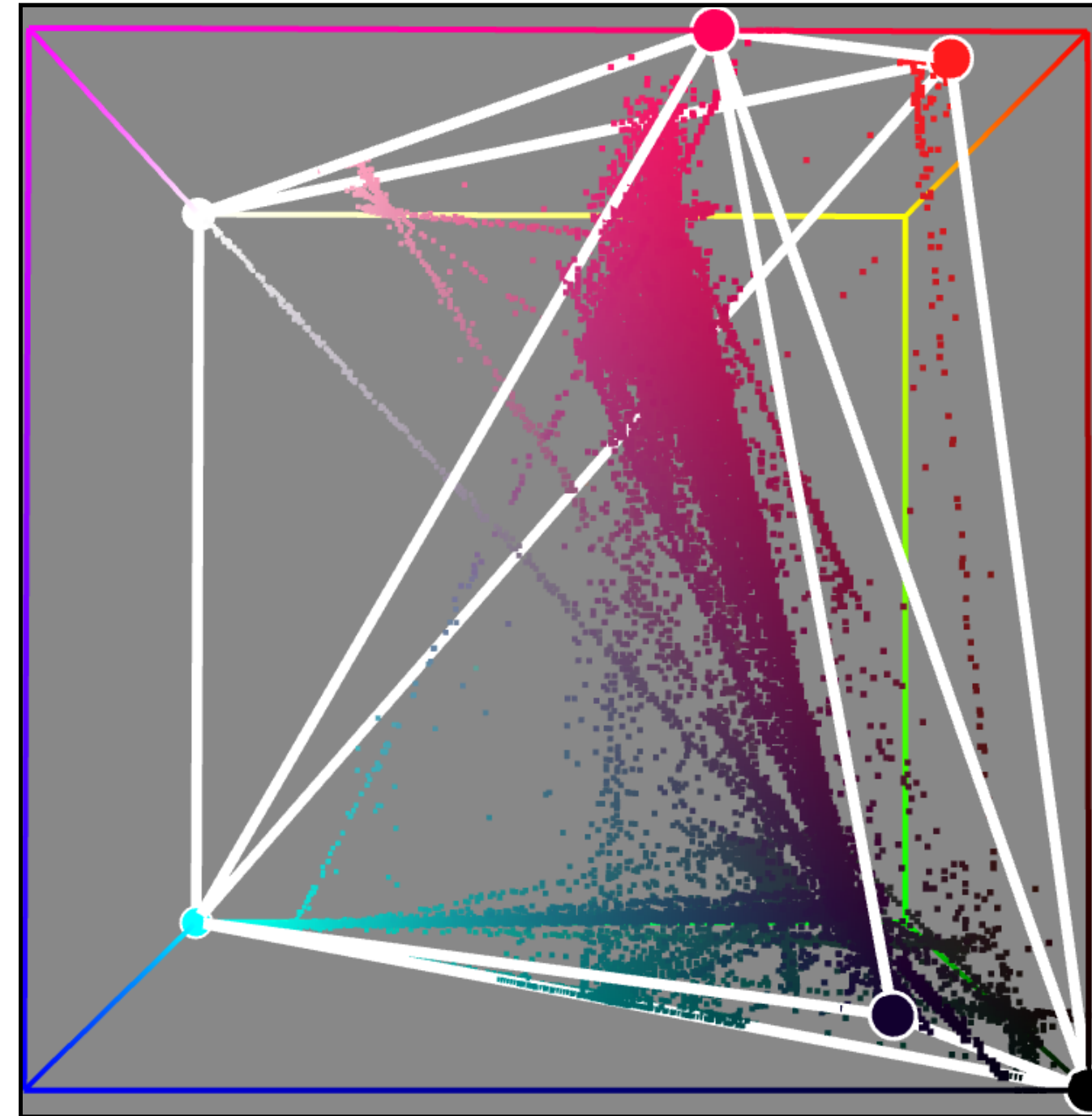




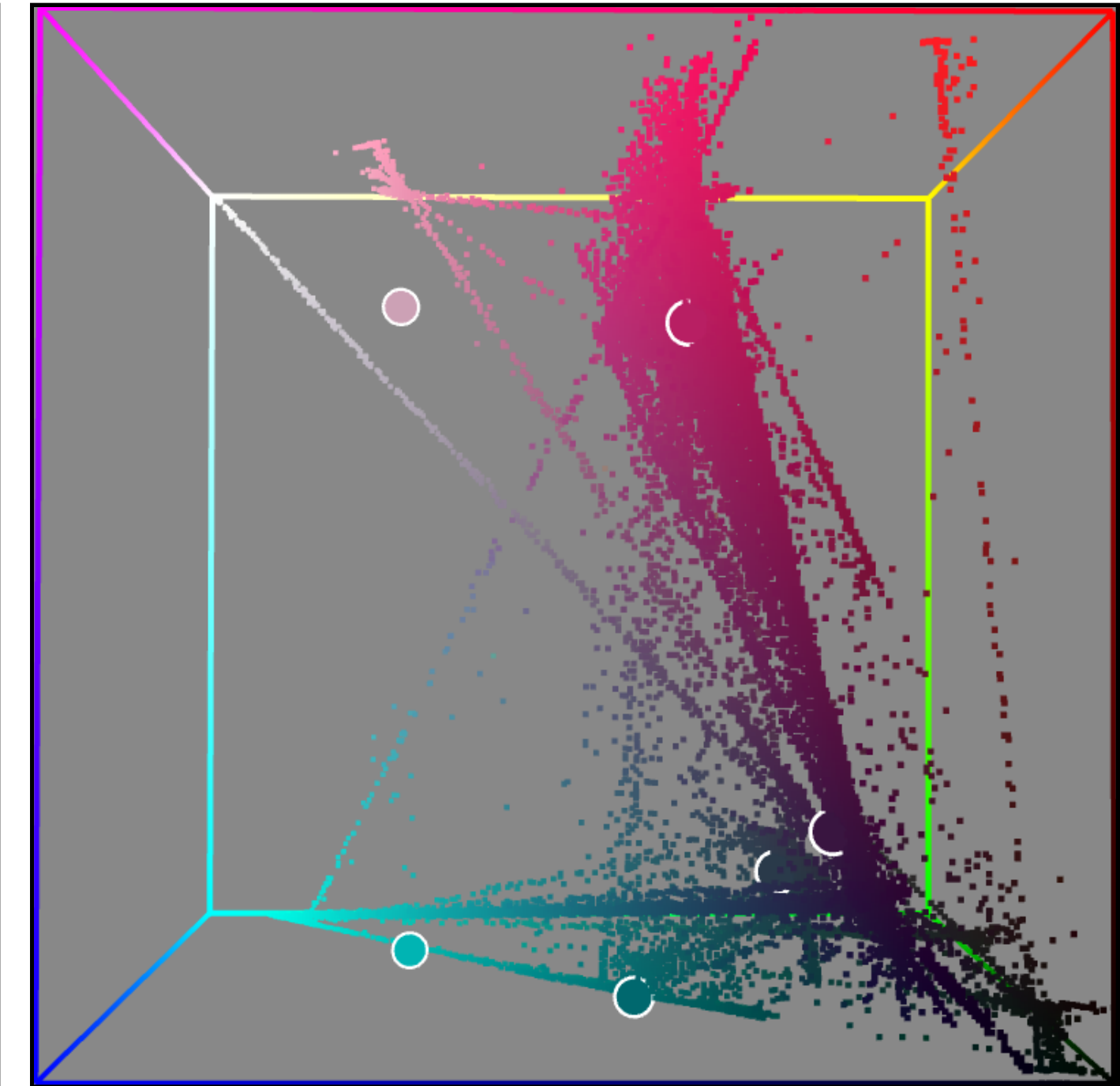
# Compared to Clustering



Input



Ours



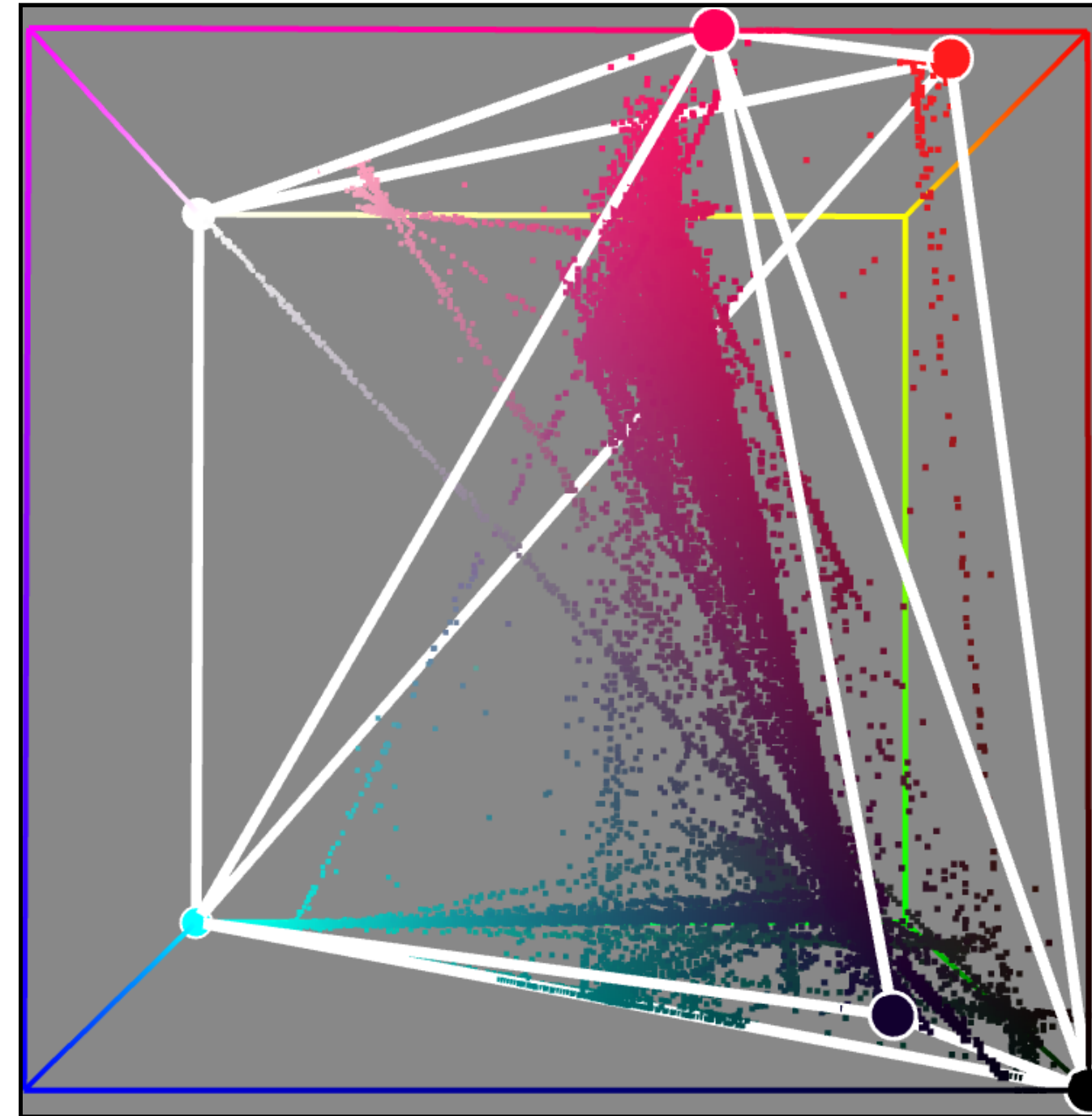
Chang et al. 2015



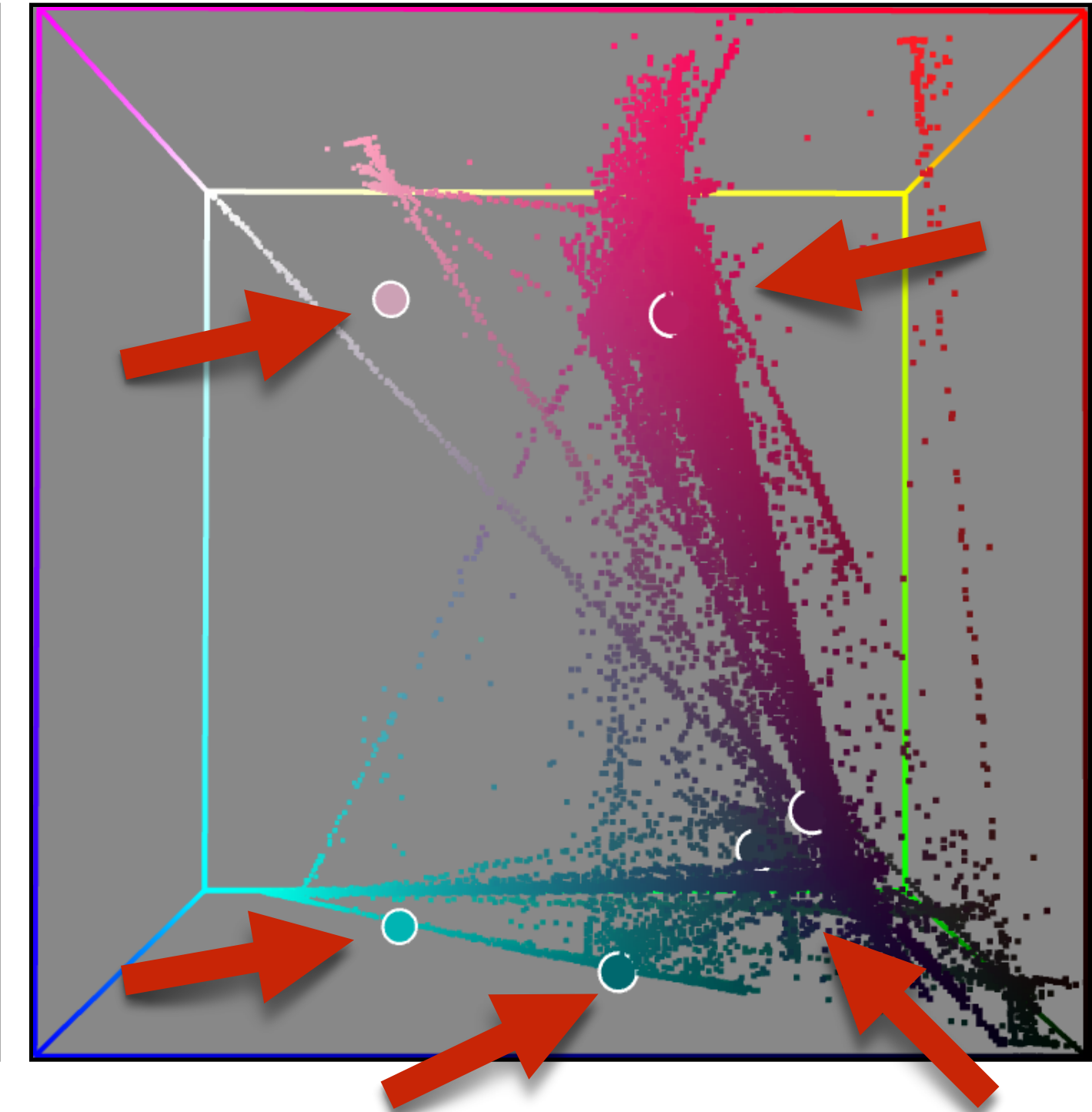
# Compared to Clustering



Input



Ours



Chang et al. 2015

Layer Opacity

# Layer Order

input



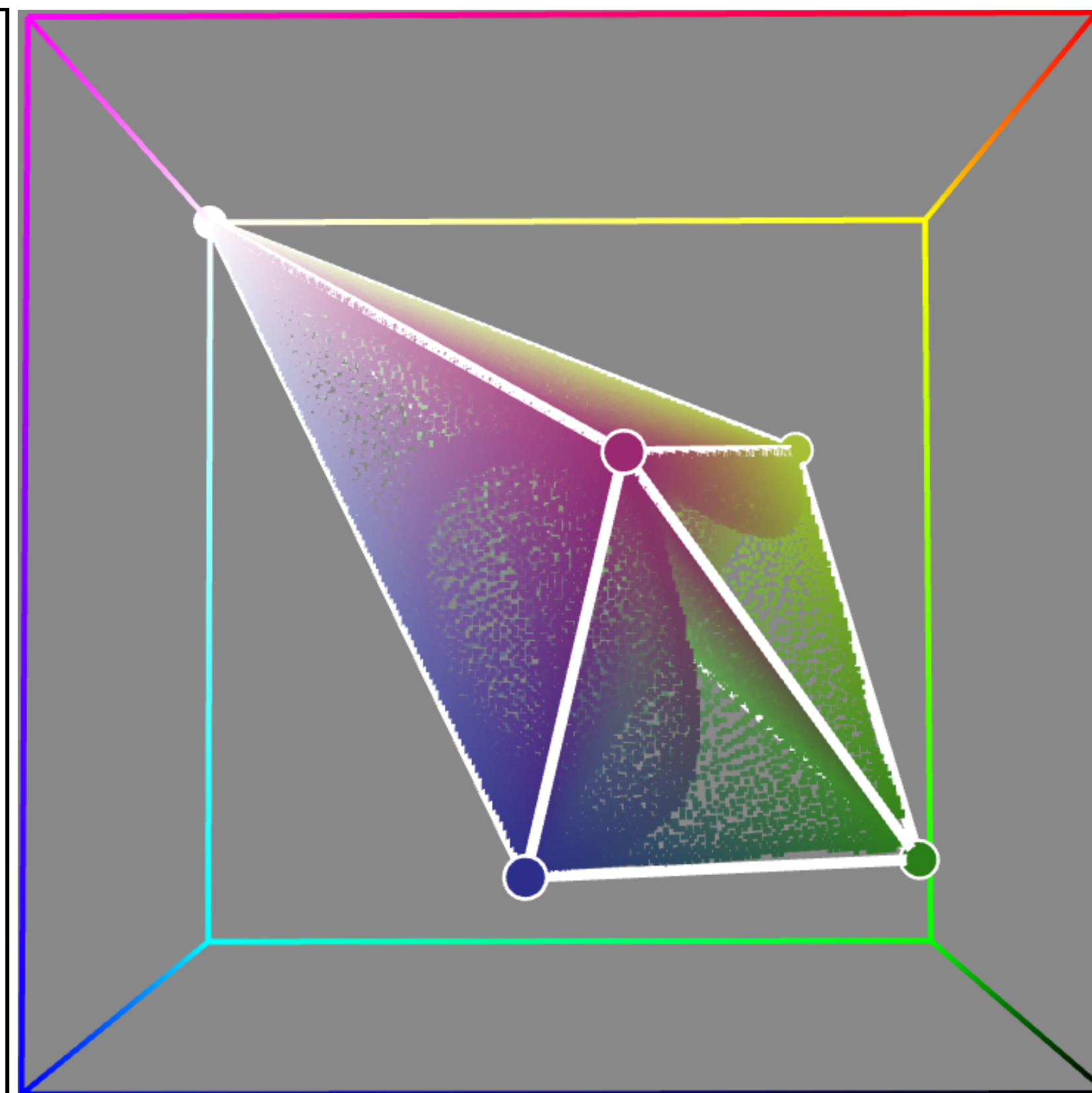


# Layer Order

input



palette selection

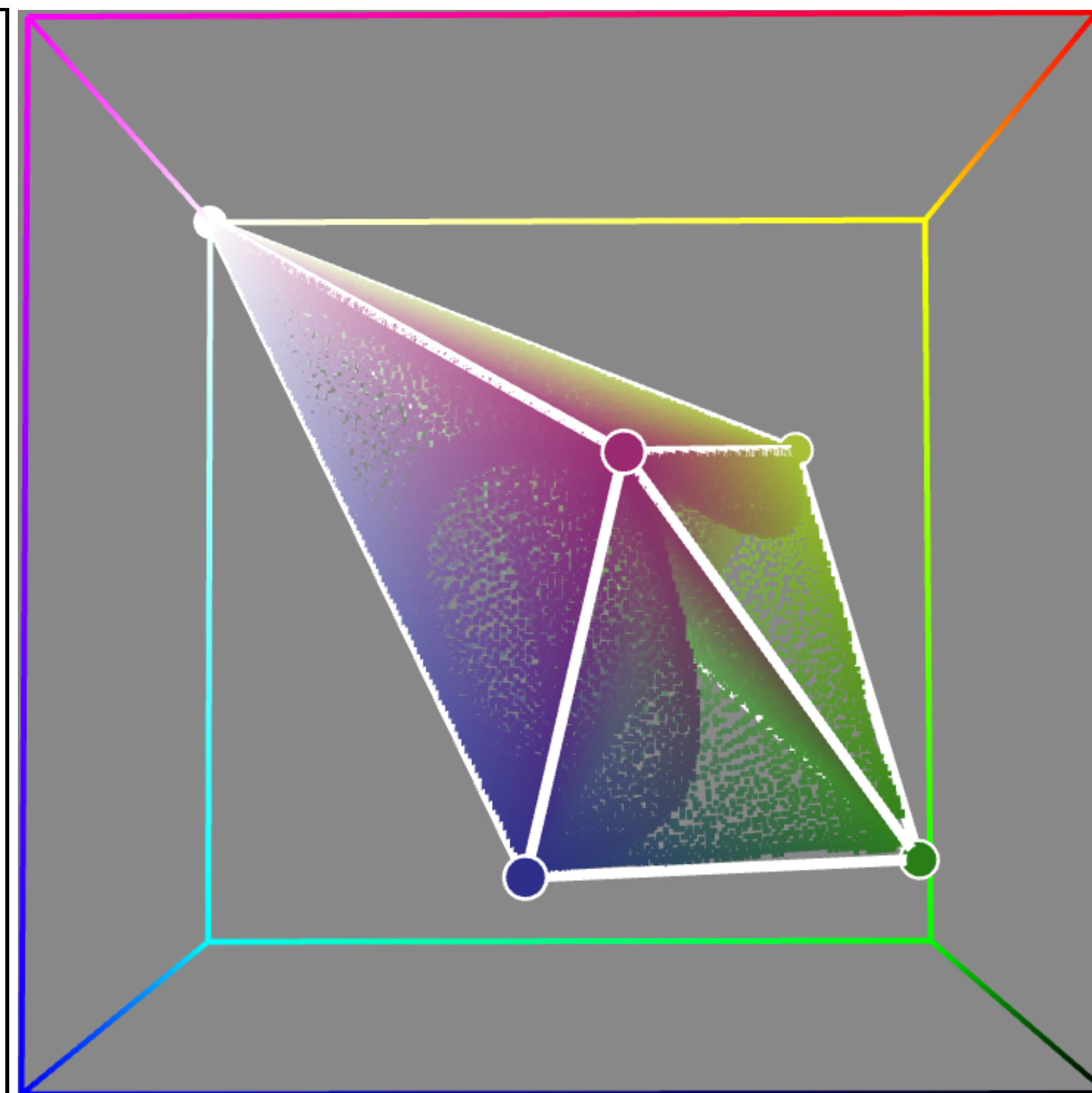


# Layer Order

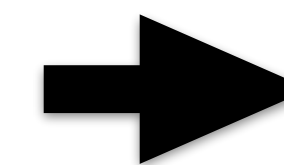
input



palette selection



palette order



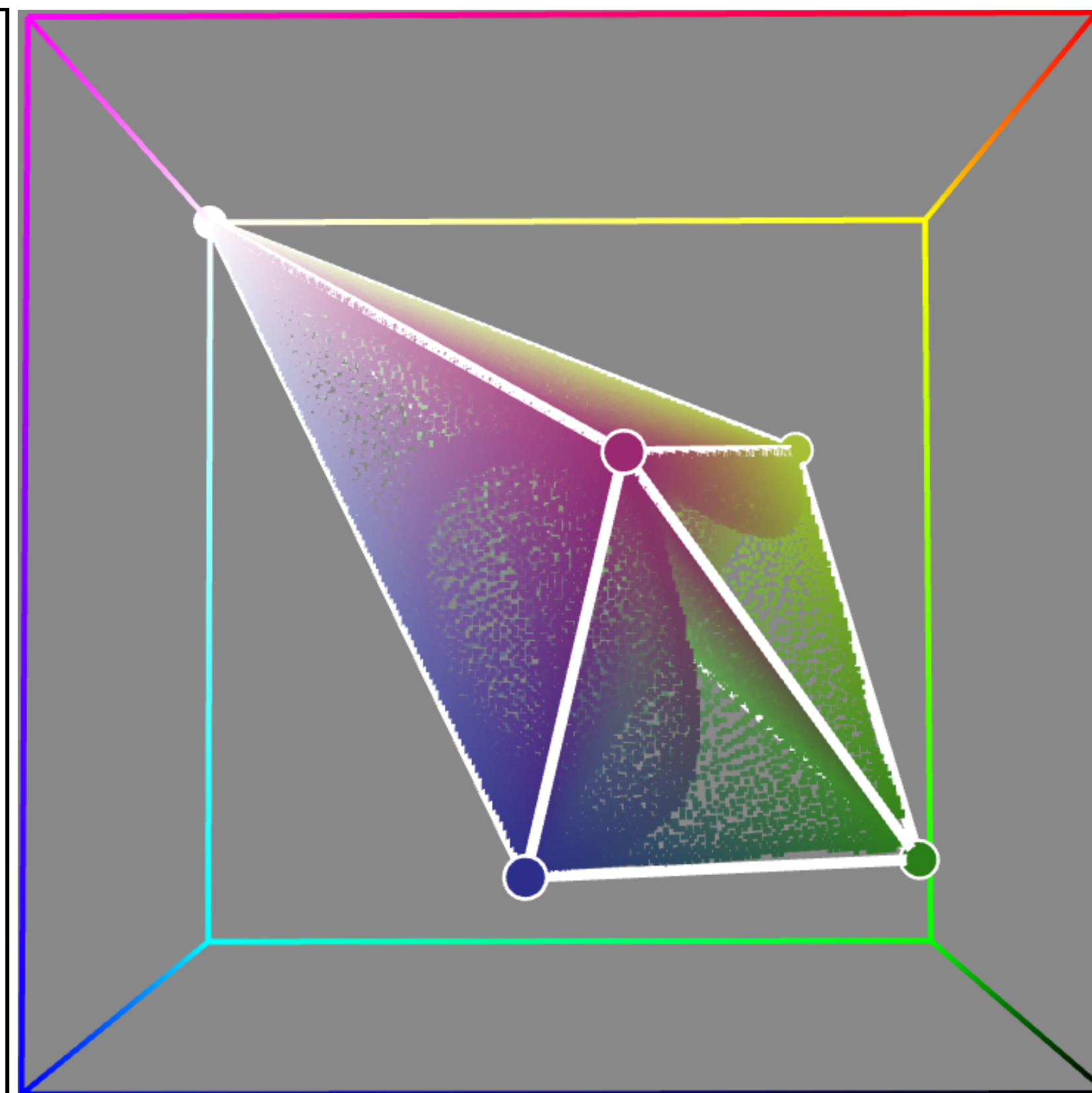


# Layer Order

input

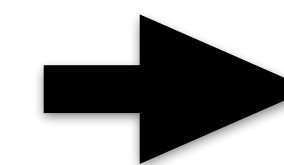
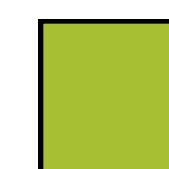
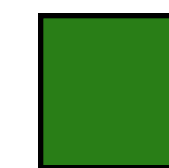
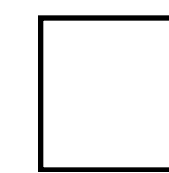


palette selection



palette order

order1



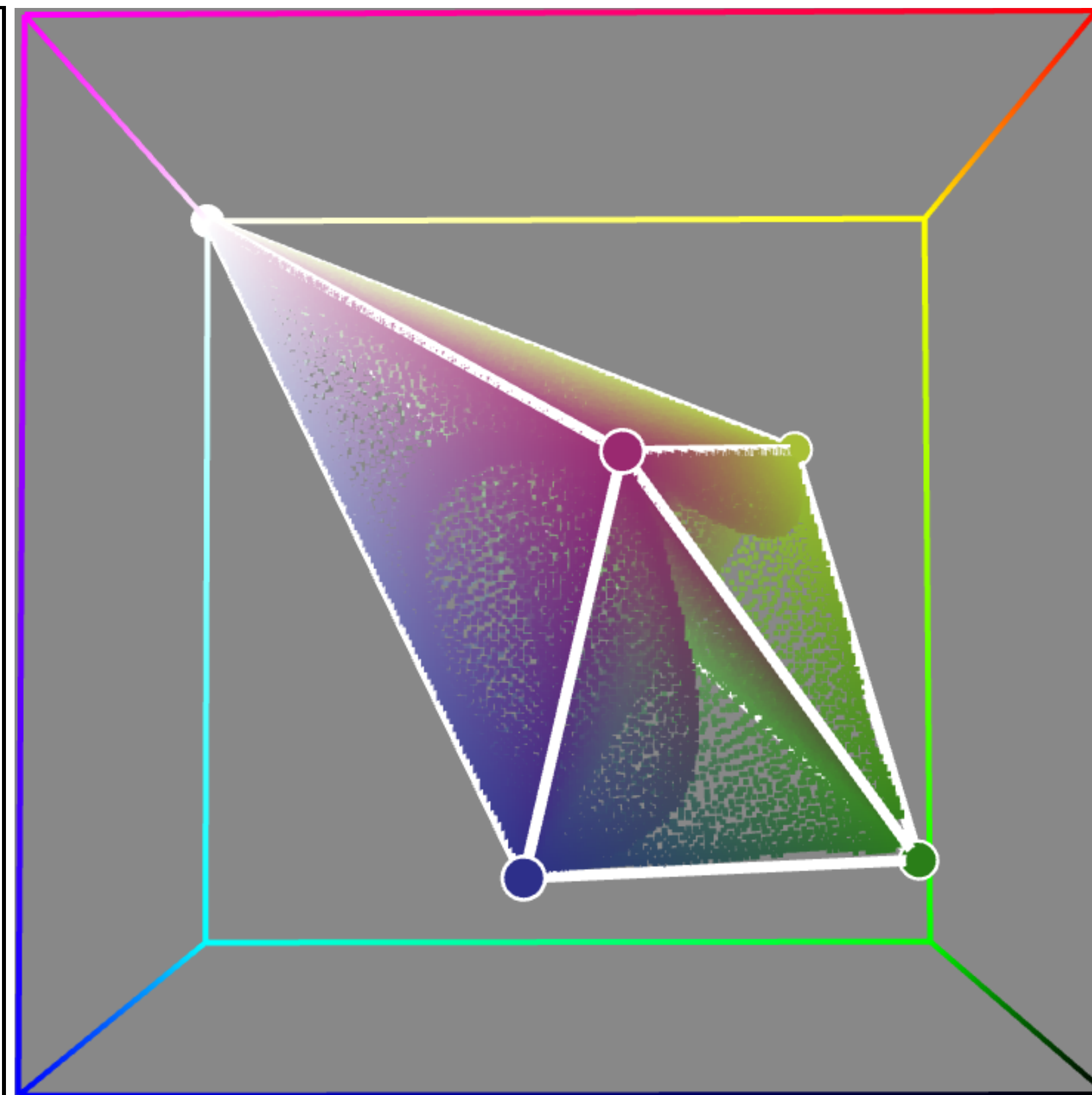


# Layer Order

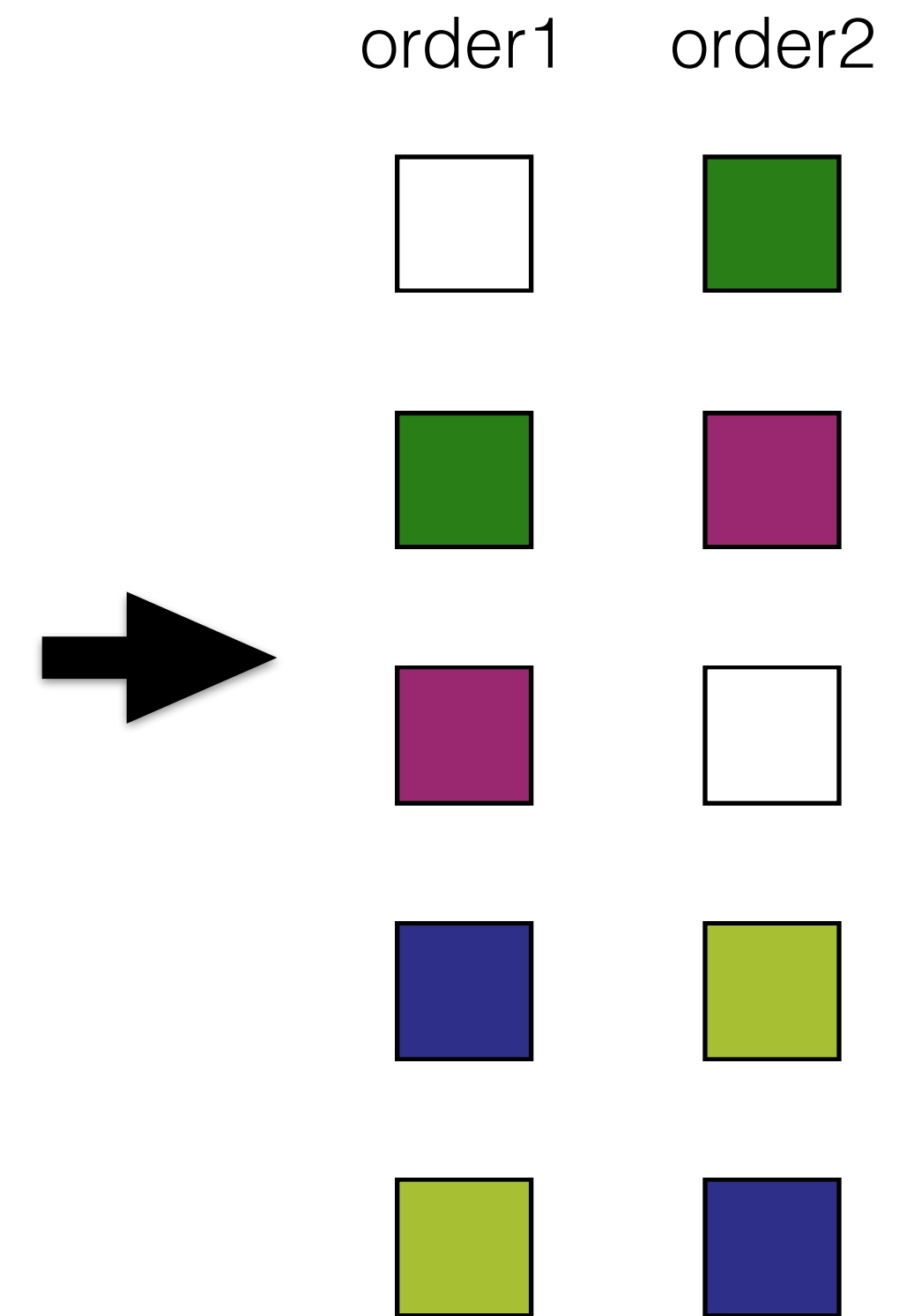
input



palette selection



palette order

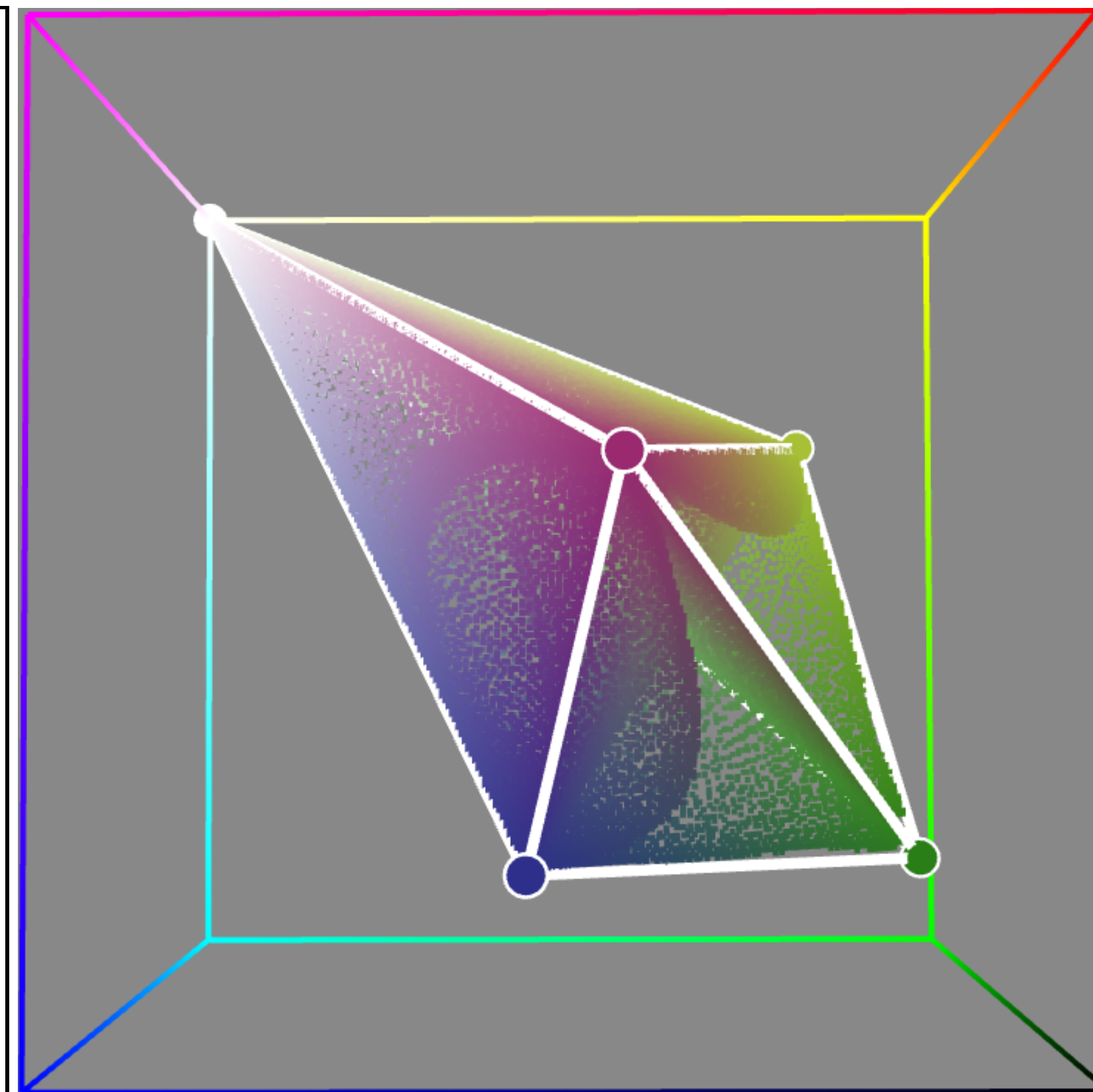


# Layer Order

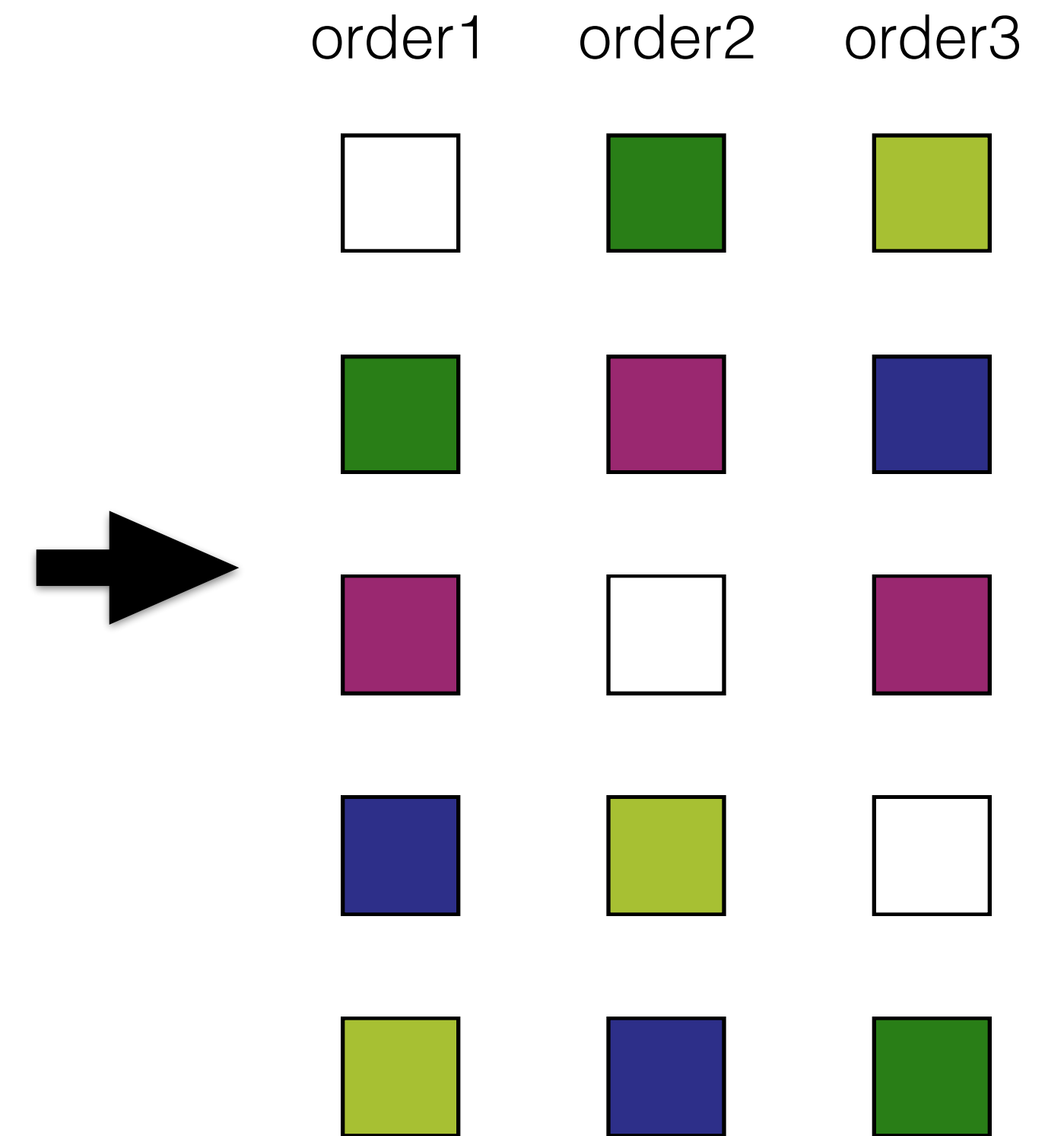
input



palette selection



palette order



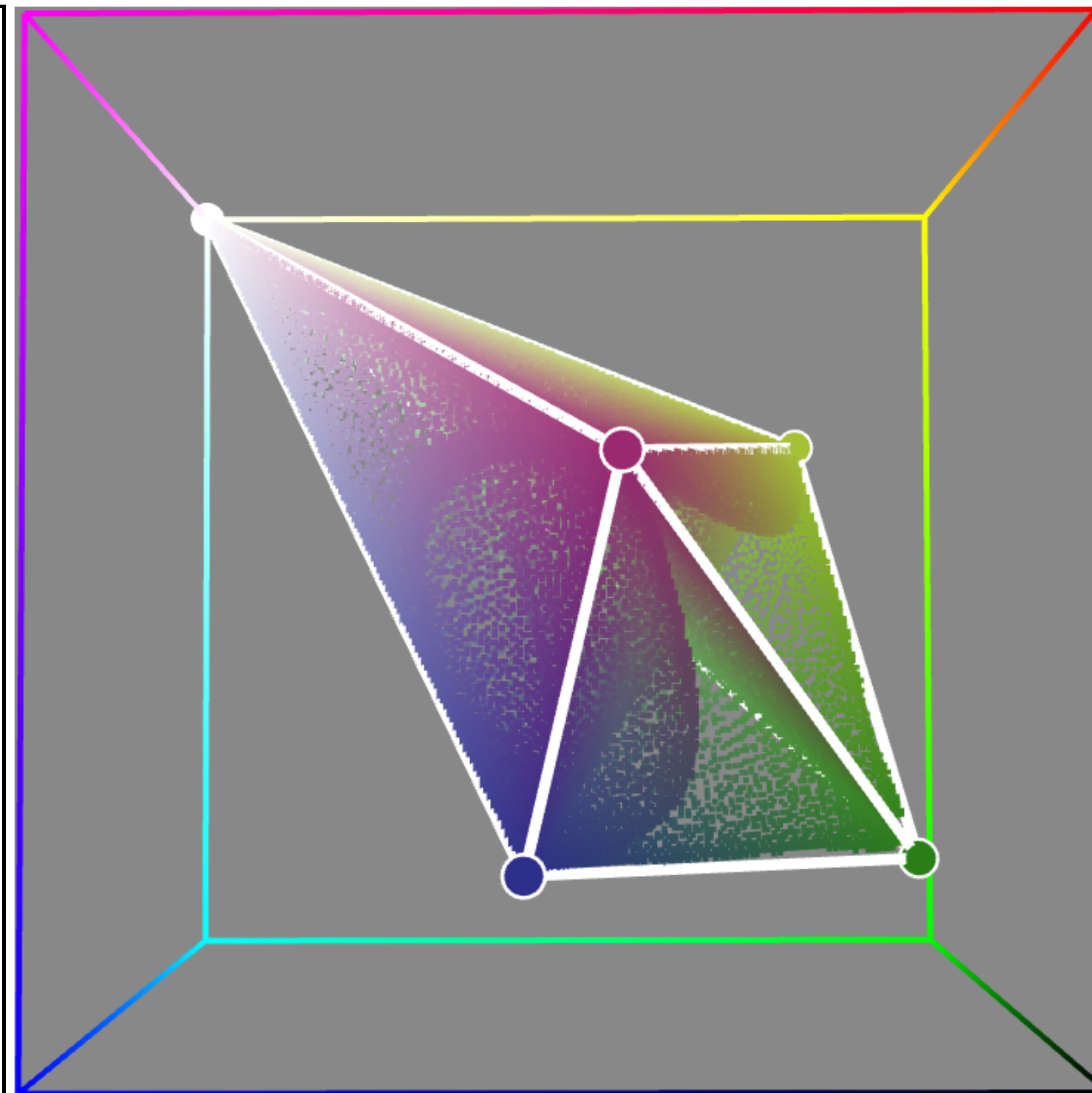


# Layer Order

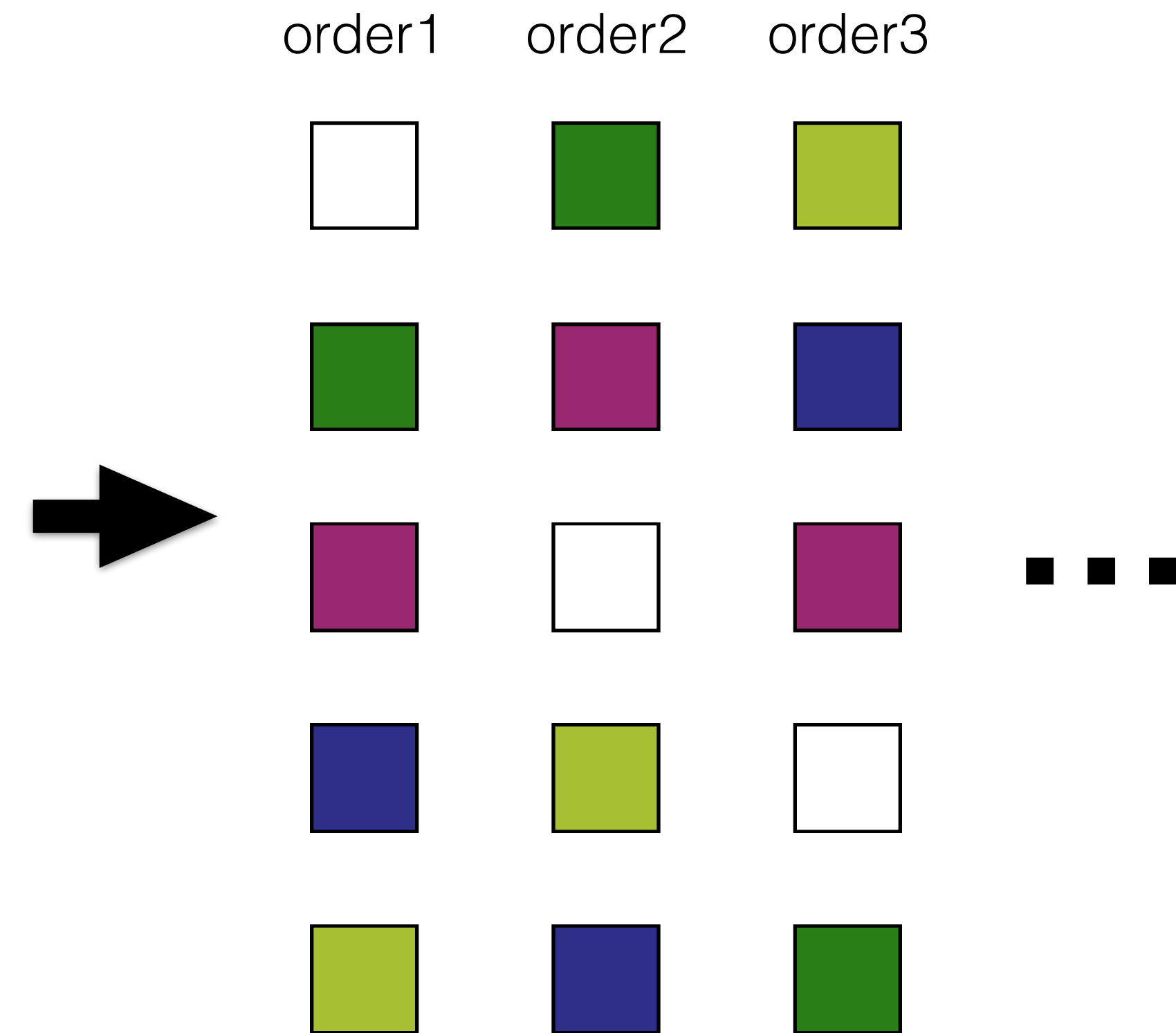
input



palette selection



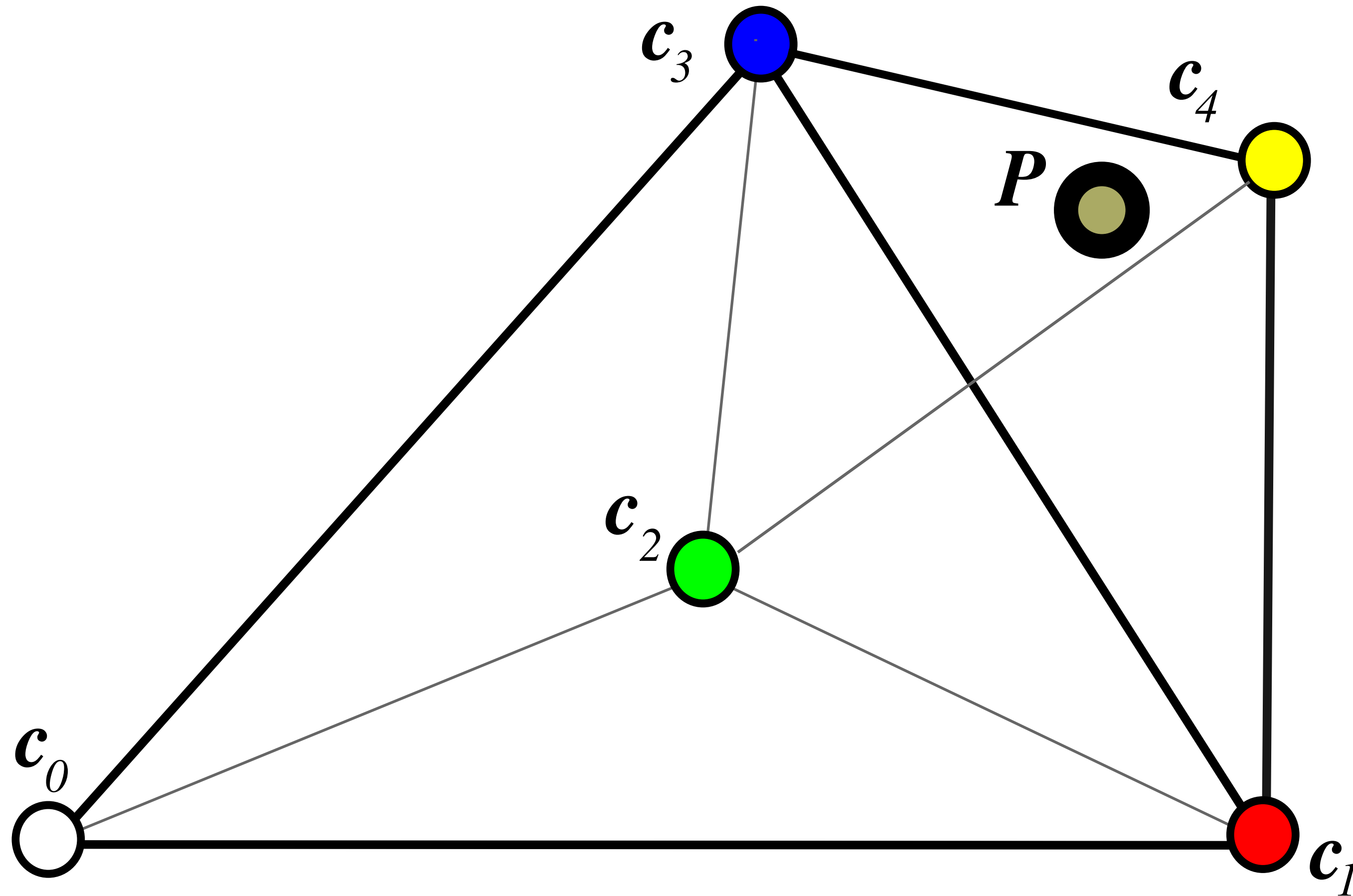
palette order





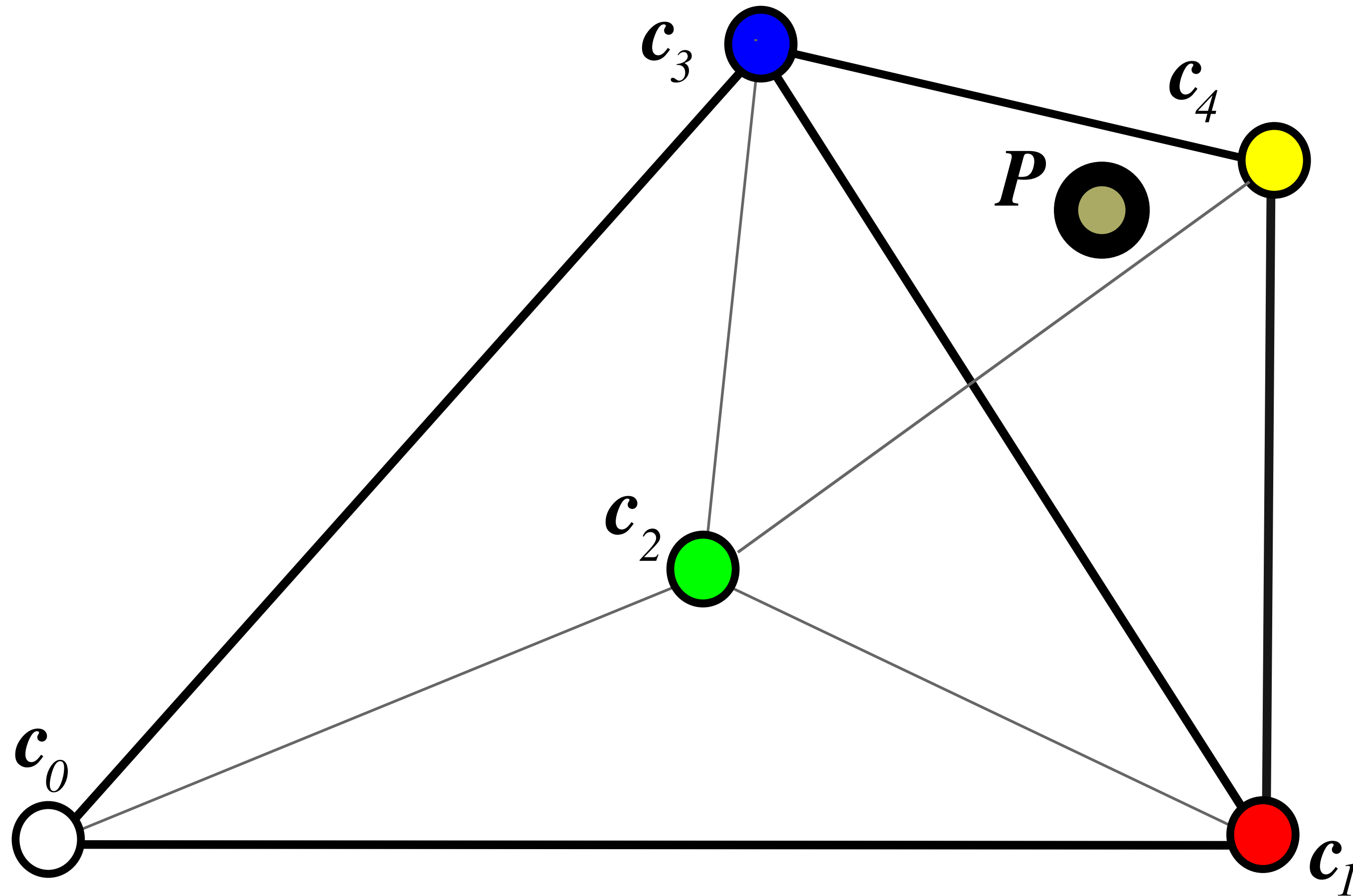
# Layer Order

- Alpha compositing is not commutative



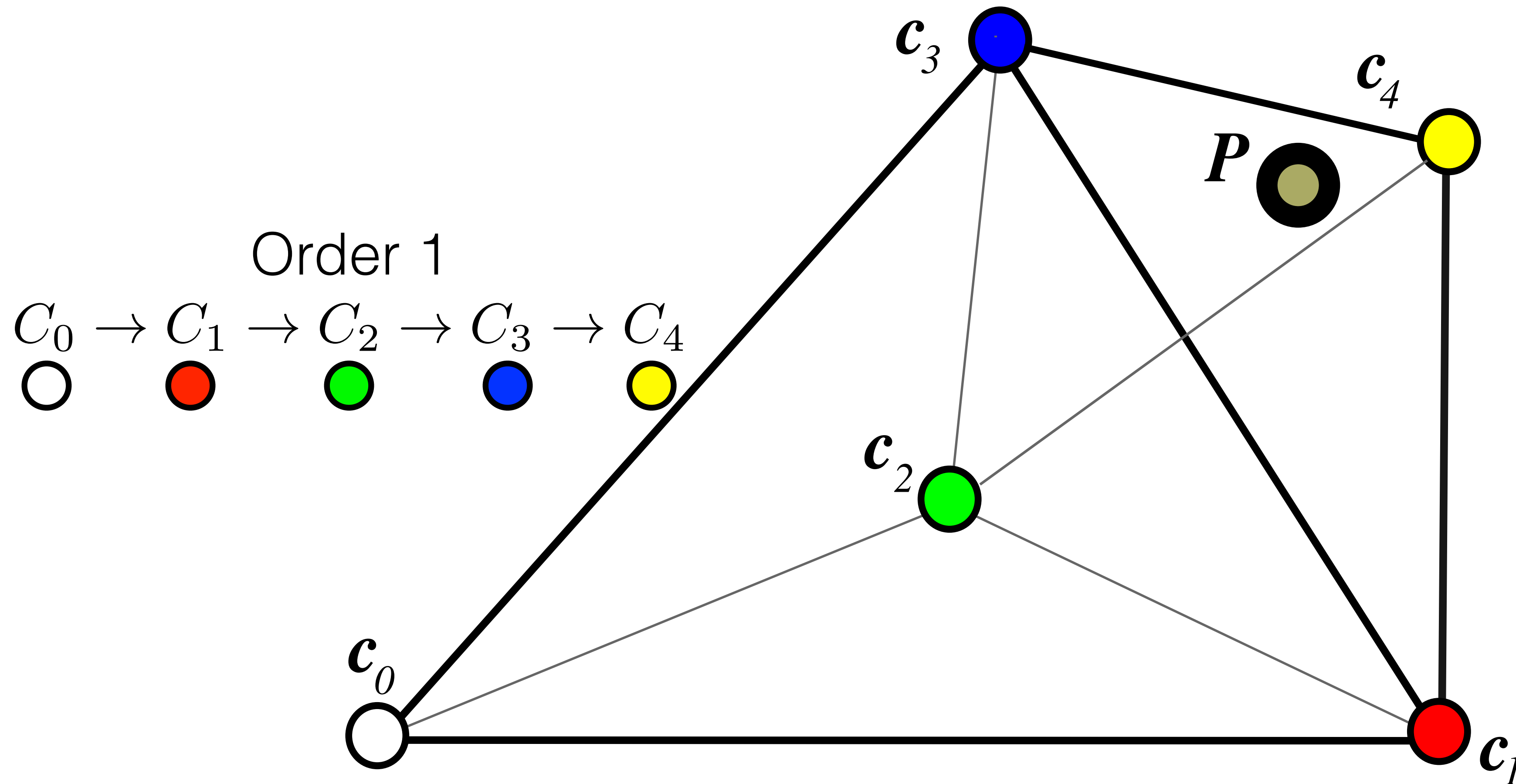
# Layer Order

- Alpha compositing is not commutative
- For  $n$  layers, there are  $n!$  orderings



# Layer Order

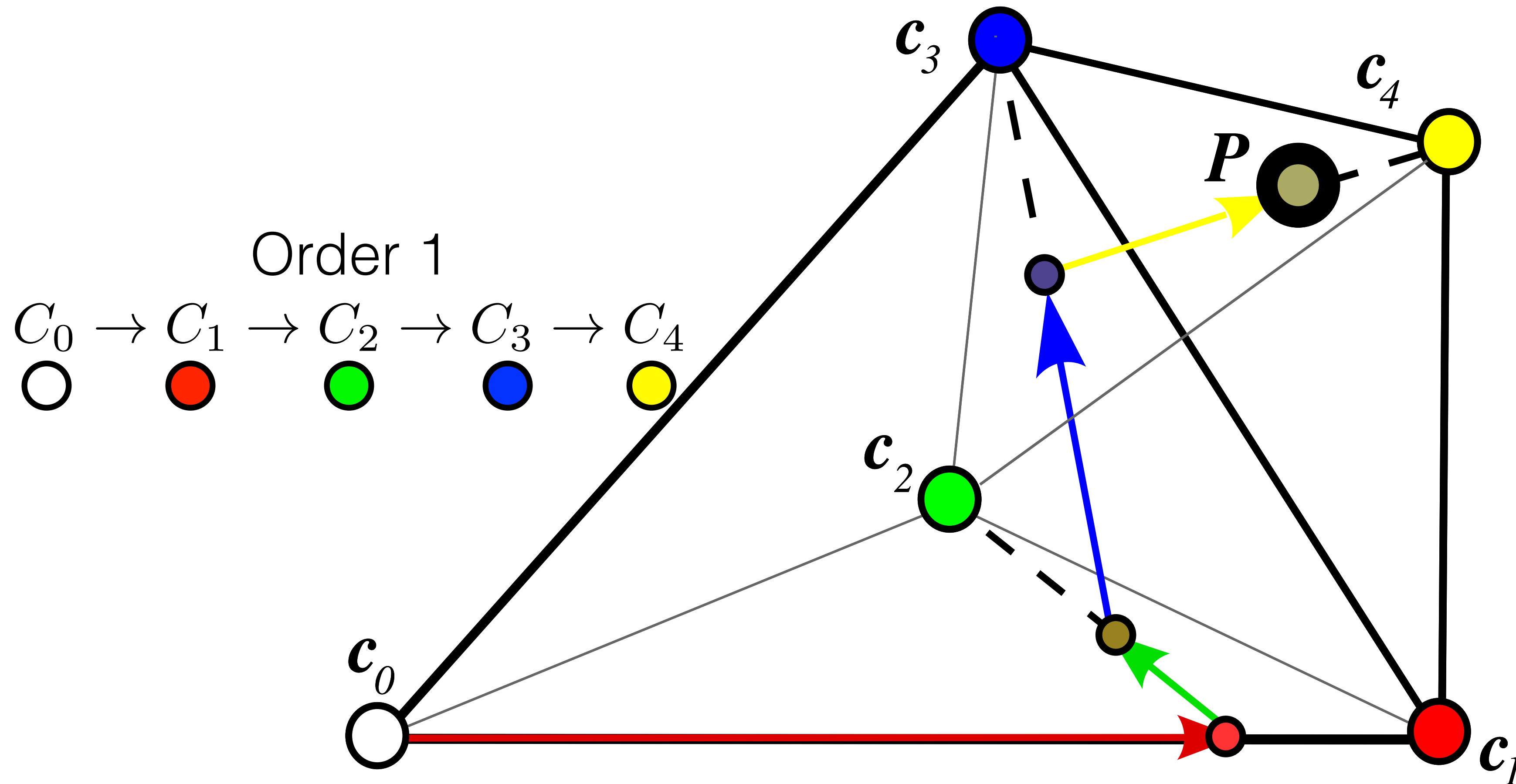
- Alpha compositing is not commutative
- For **n** layers, there are **n!** orderings





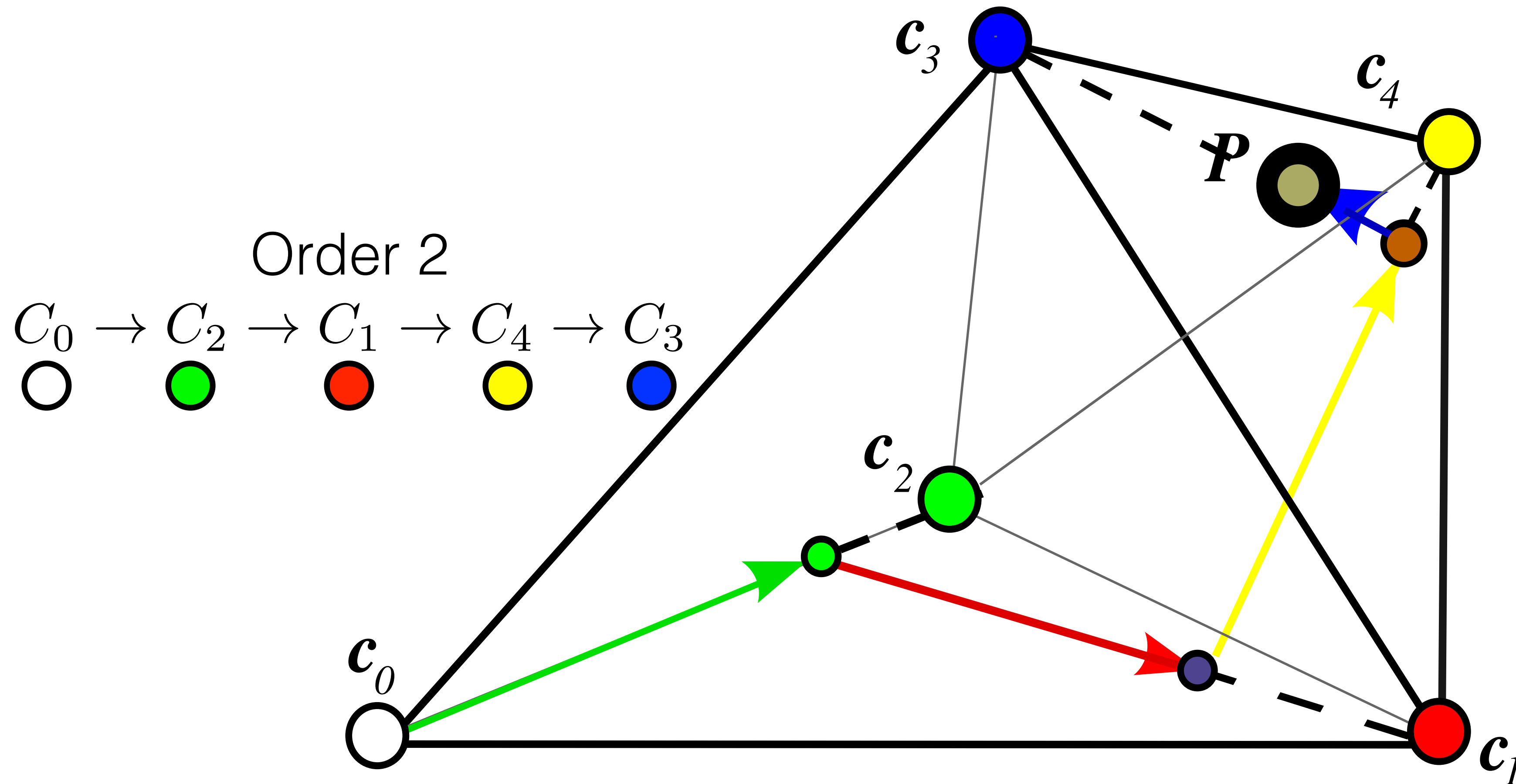
# Layer Order

- Alpha compositing is not commutative
- For  $n$  layers, there are  $n!$  orderings



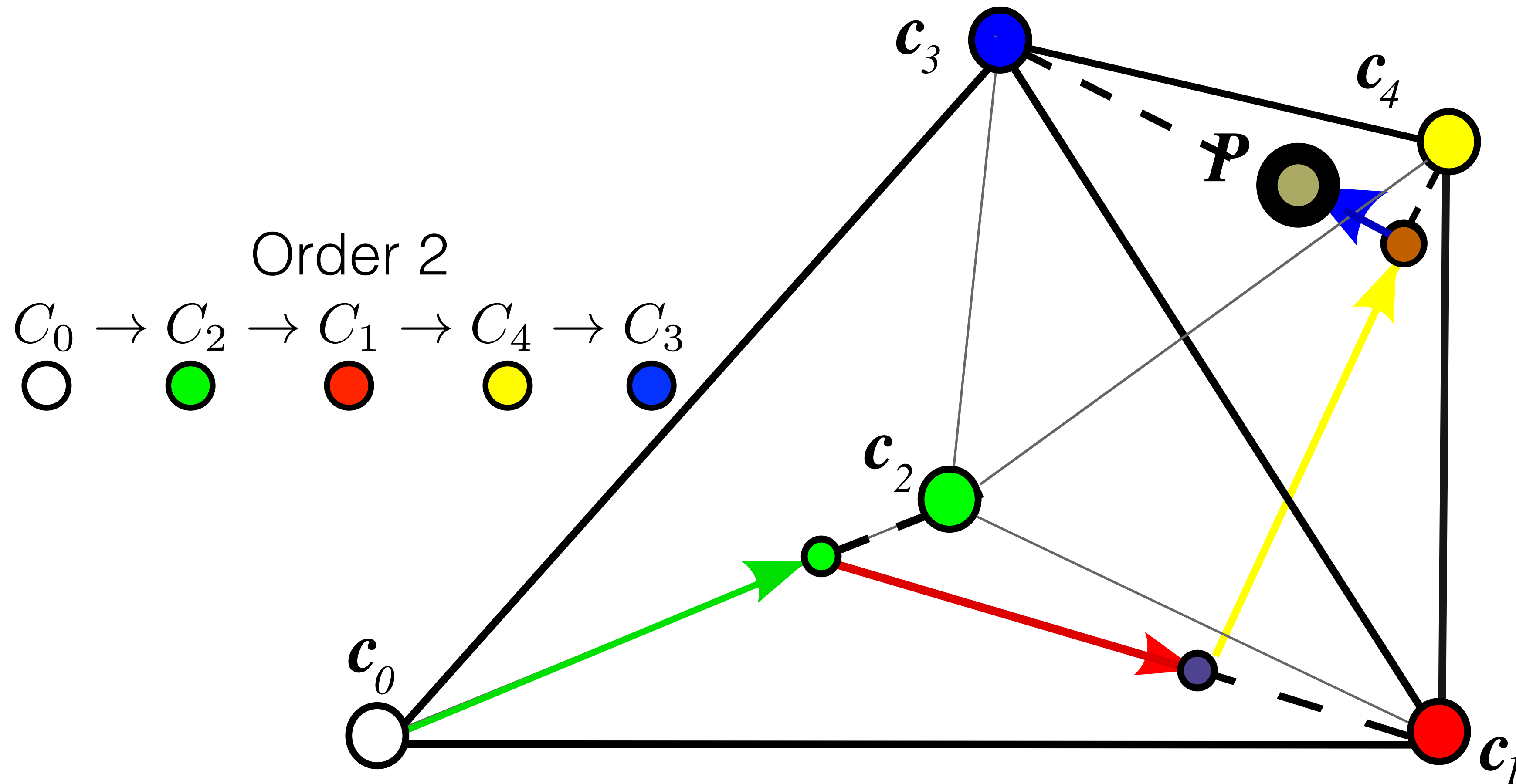
# Layer Order

- Alpha compositing is not commutative
- For  $n$  layers, there are  $n!$  orderings



# Layer Order

- Alpha compositing is not commutative
- For  $n$  layers, there are  $n!$  orderings
- **Hard to find good metric.**
- **User manually chooses.**



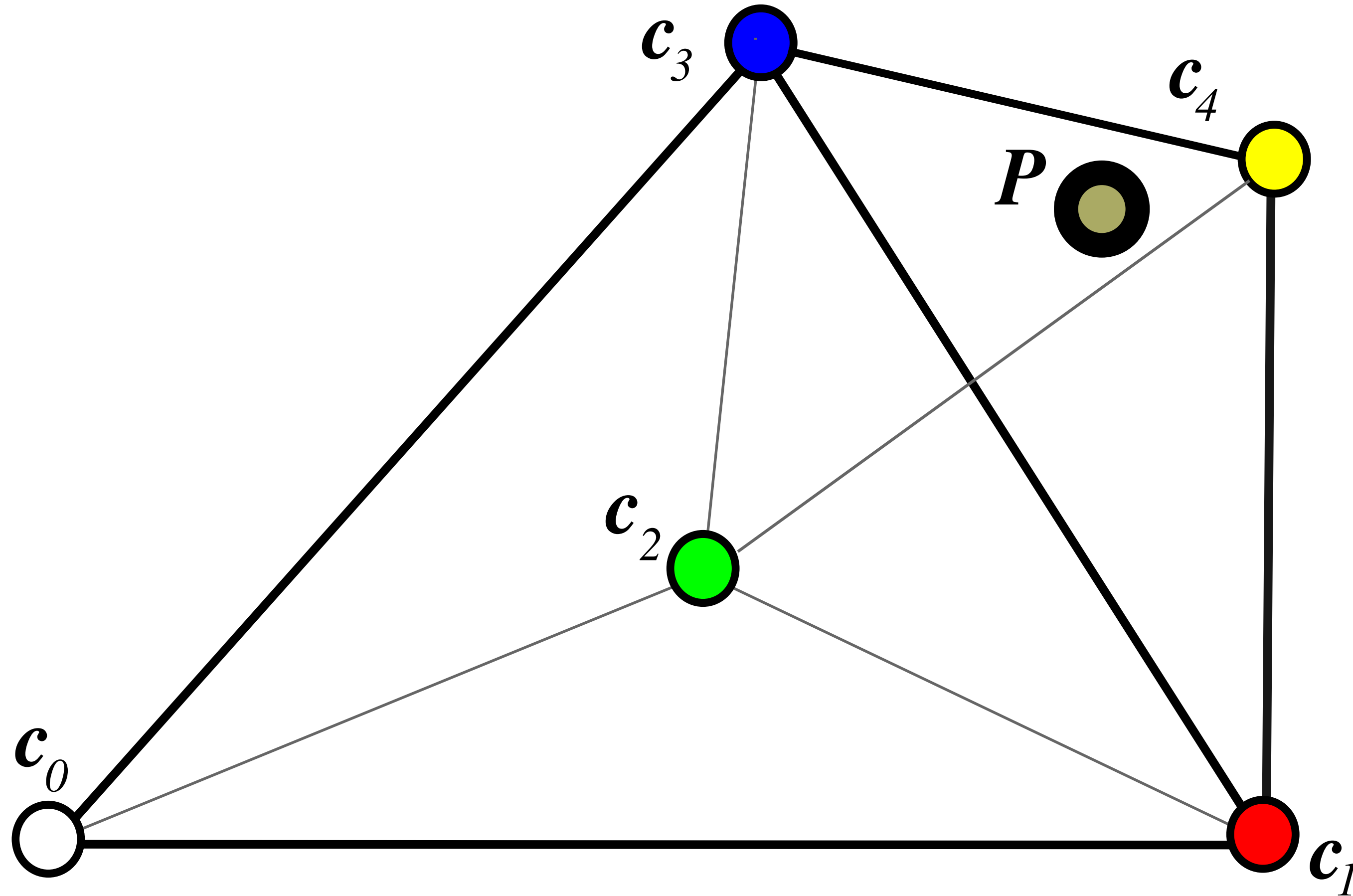


# Color Compositing Path

- After user chooses a layer order:  $\overset{\circ}{C_0} \rightarrow \overset{\bullet}{C_1} \rightarrow \overset{\bullet}{C_2} \rightarrow \overset{\bullet}{C_3} \rightarrow \overset{\bullet}{C_4}$
- Still have **infinite** paths from  $C_0$  to  $P$

# Color Compositing Path

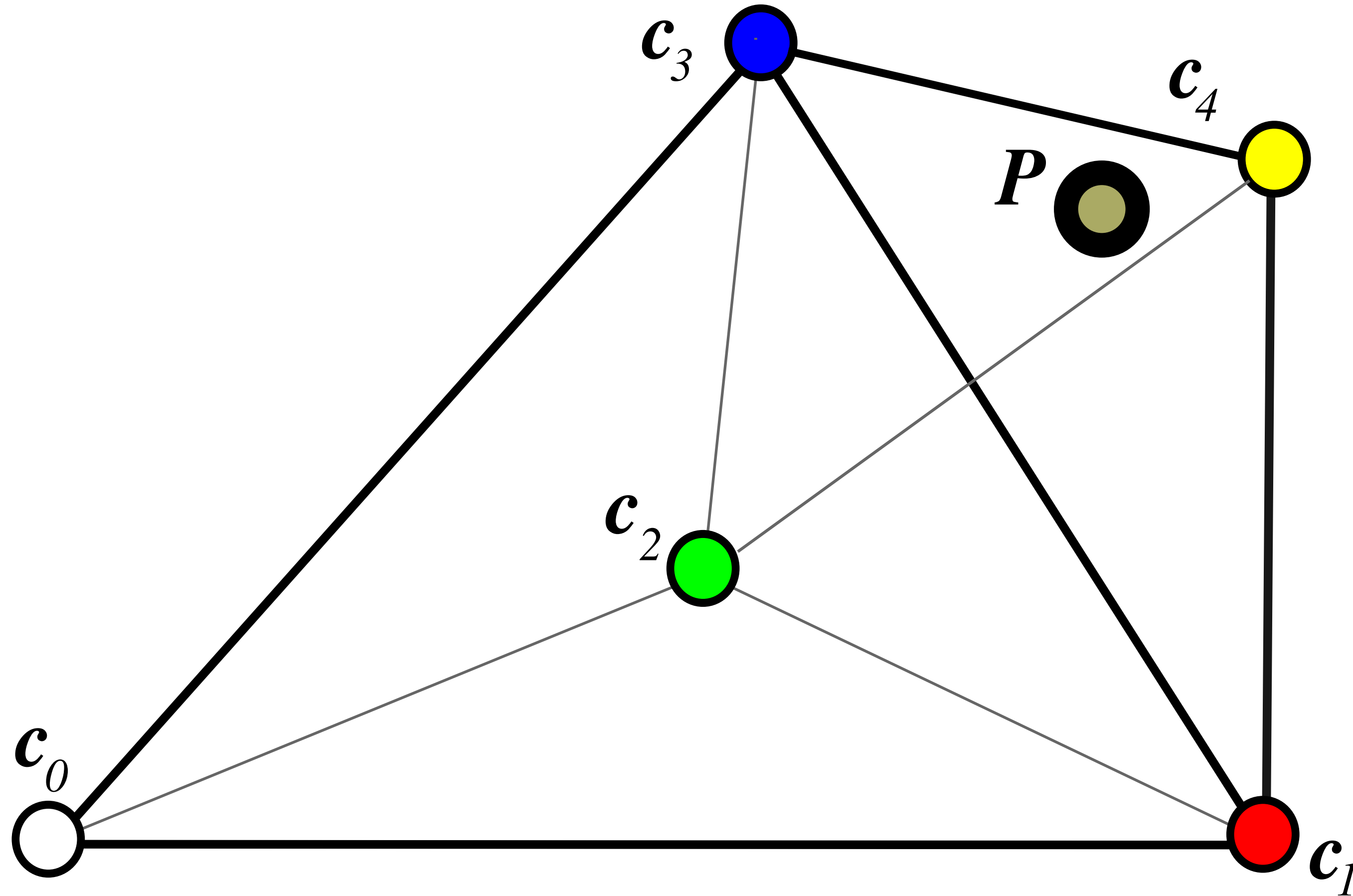
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Path 1

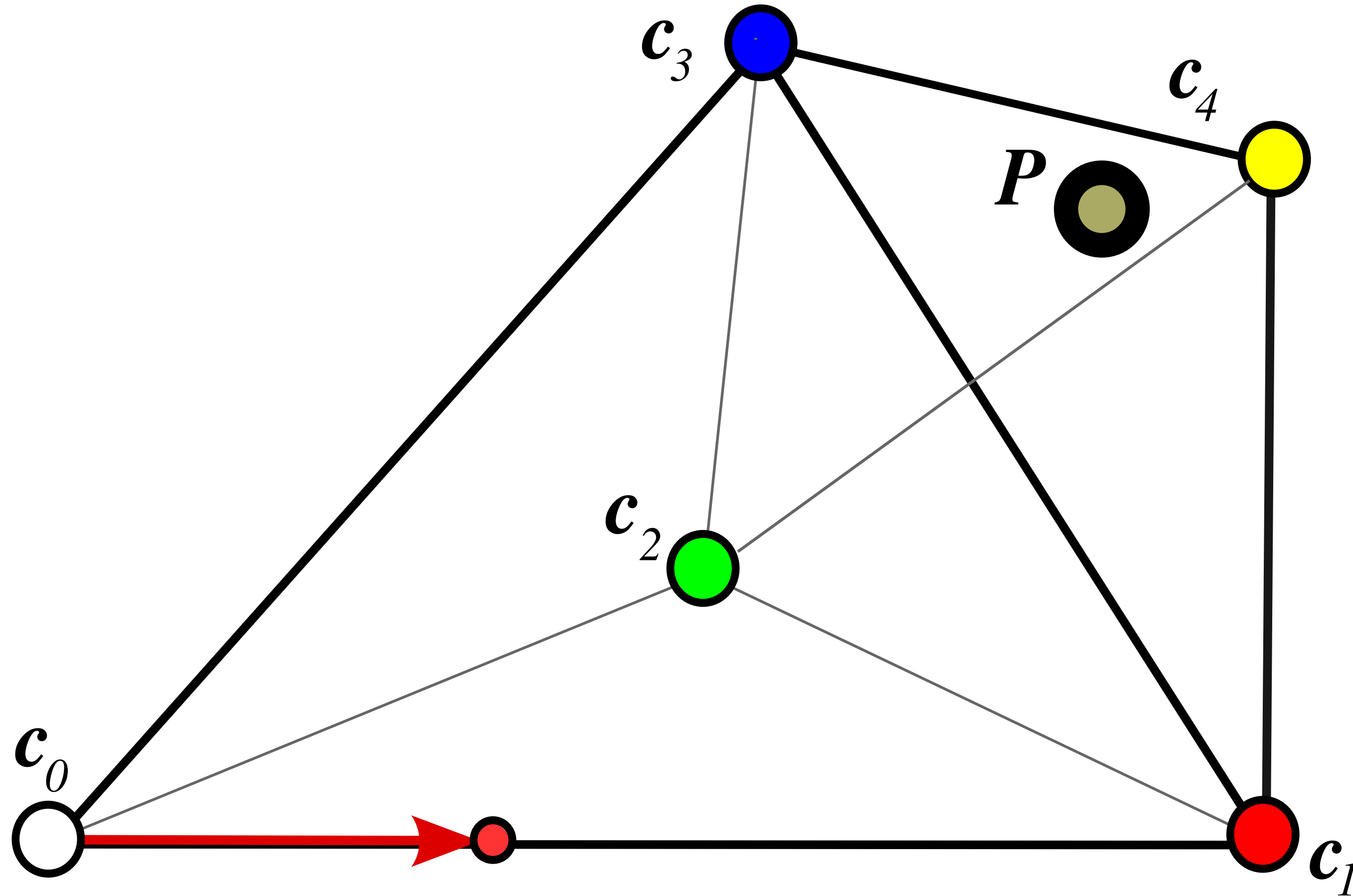




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- After user chooses a layer order:  $C_0 \rightarrow C_1 \rightarrow C_2 \rightarrow C_3 \rightarrow C_4$
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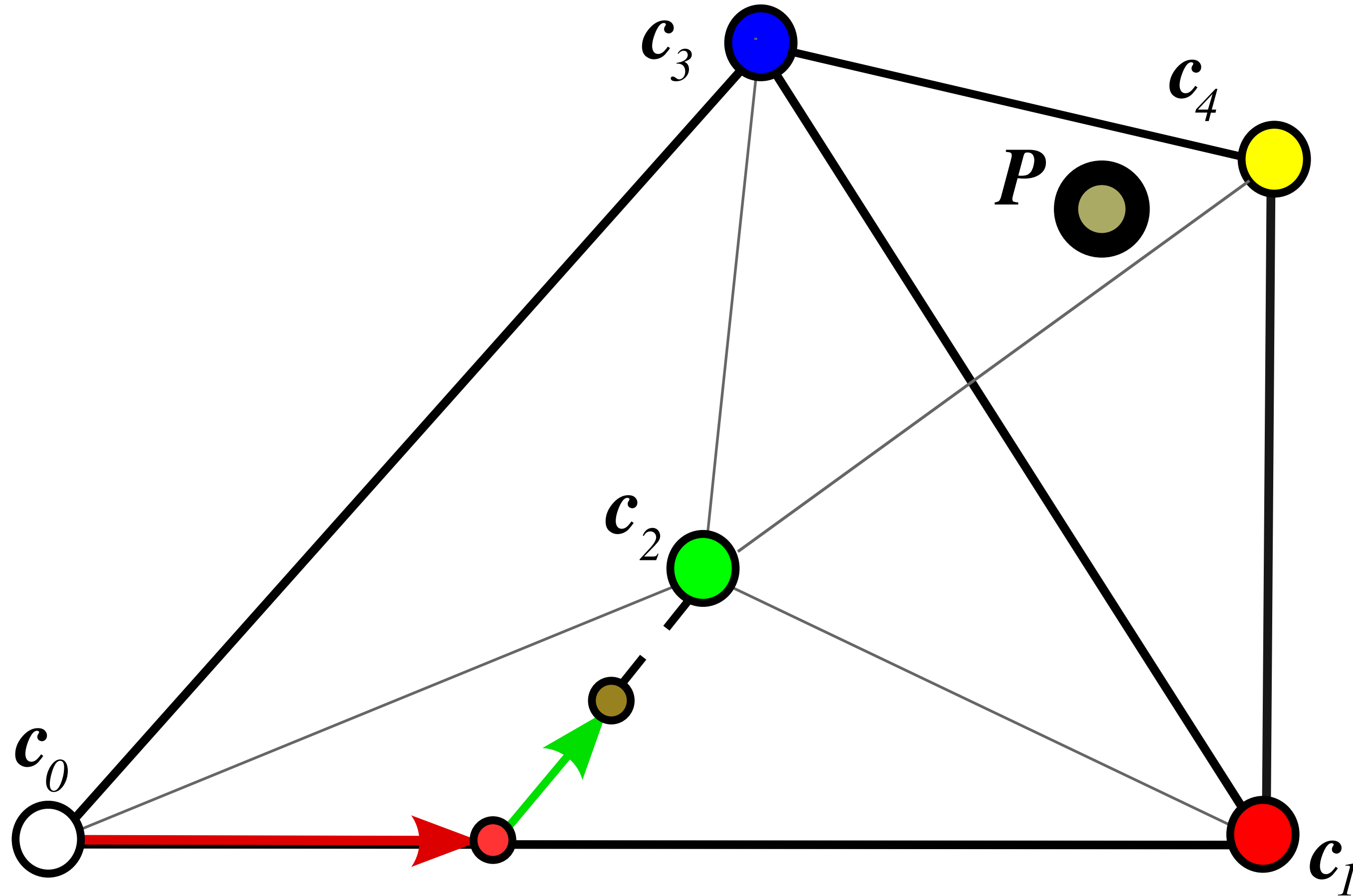
Path 1



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- After user chooses a layer order:  $C_0 \rightarrow C_1 \rightarrow C_2 \rightarrow C_3 \rightarrow C_4$
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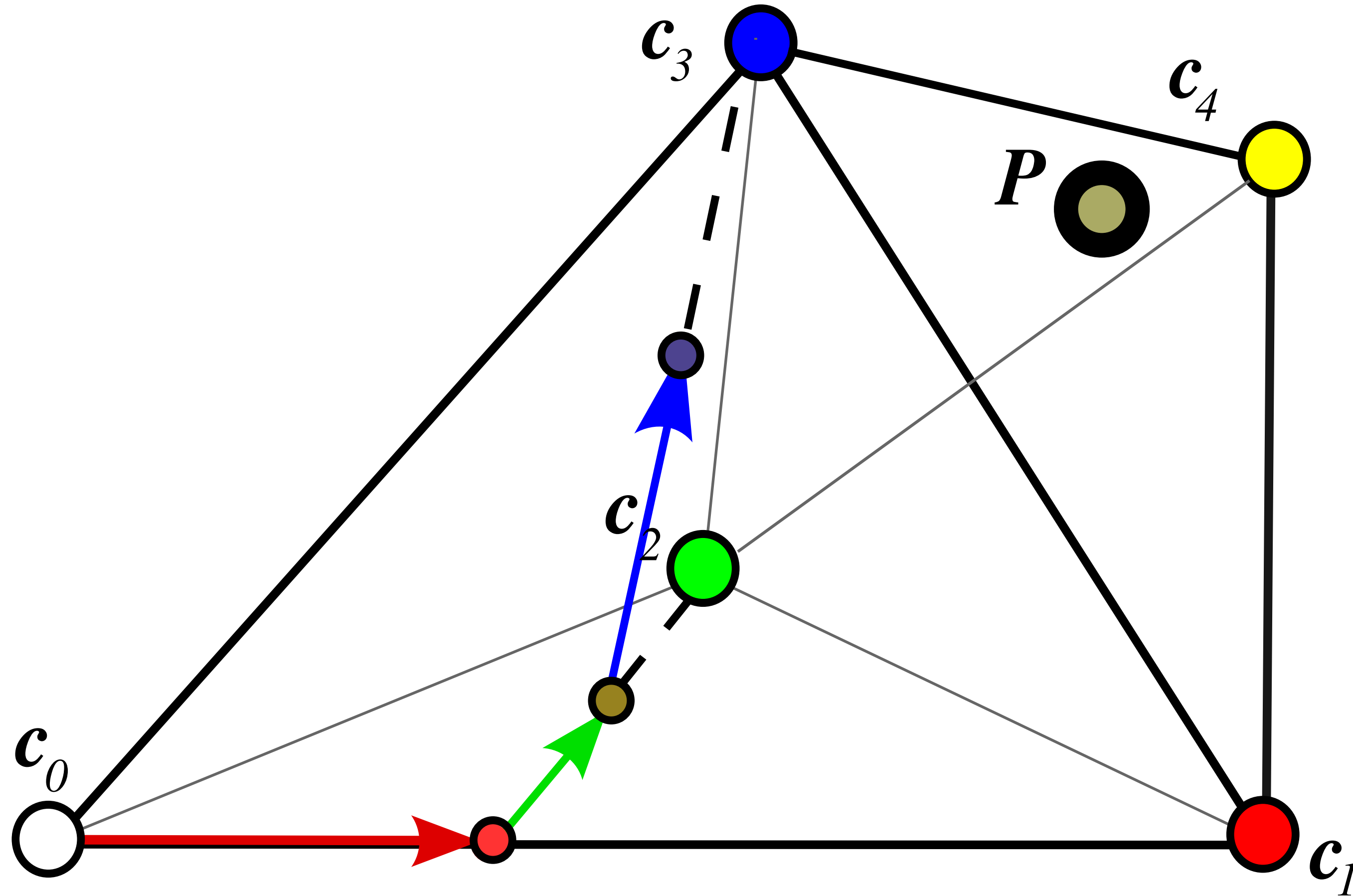
Path 1



# Color Compositing Path

- After user chooses a layer order:  $C_0 \rightarrow C_1 \rightarrow C_2 \rightarrow C_3 \rightarrow C_4$
- Still have **infinite** paths from  $C_0$  to  $P$

Path 1

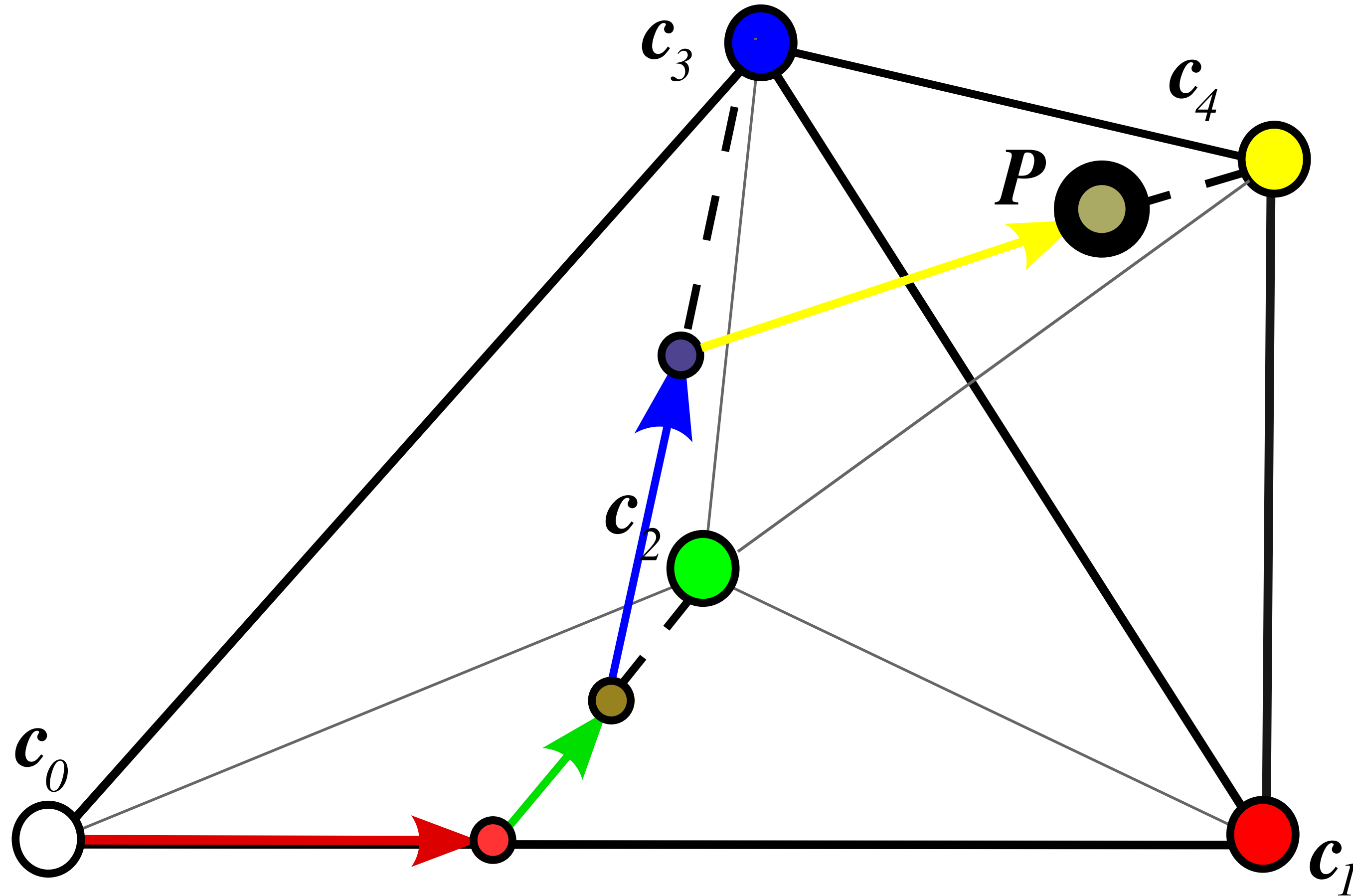




# Color Compositing Path

- After user chooses a layer order:  $C_0 \rightarrow C_1 \rightarrow C_2 \rightarrow C_3 \rightarrow C_4$
- Still have **infinite** paths from  $C_0$  to  $P$

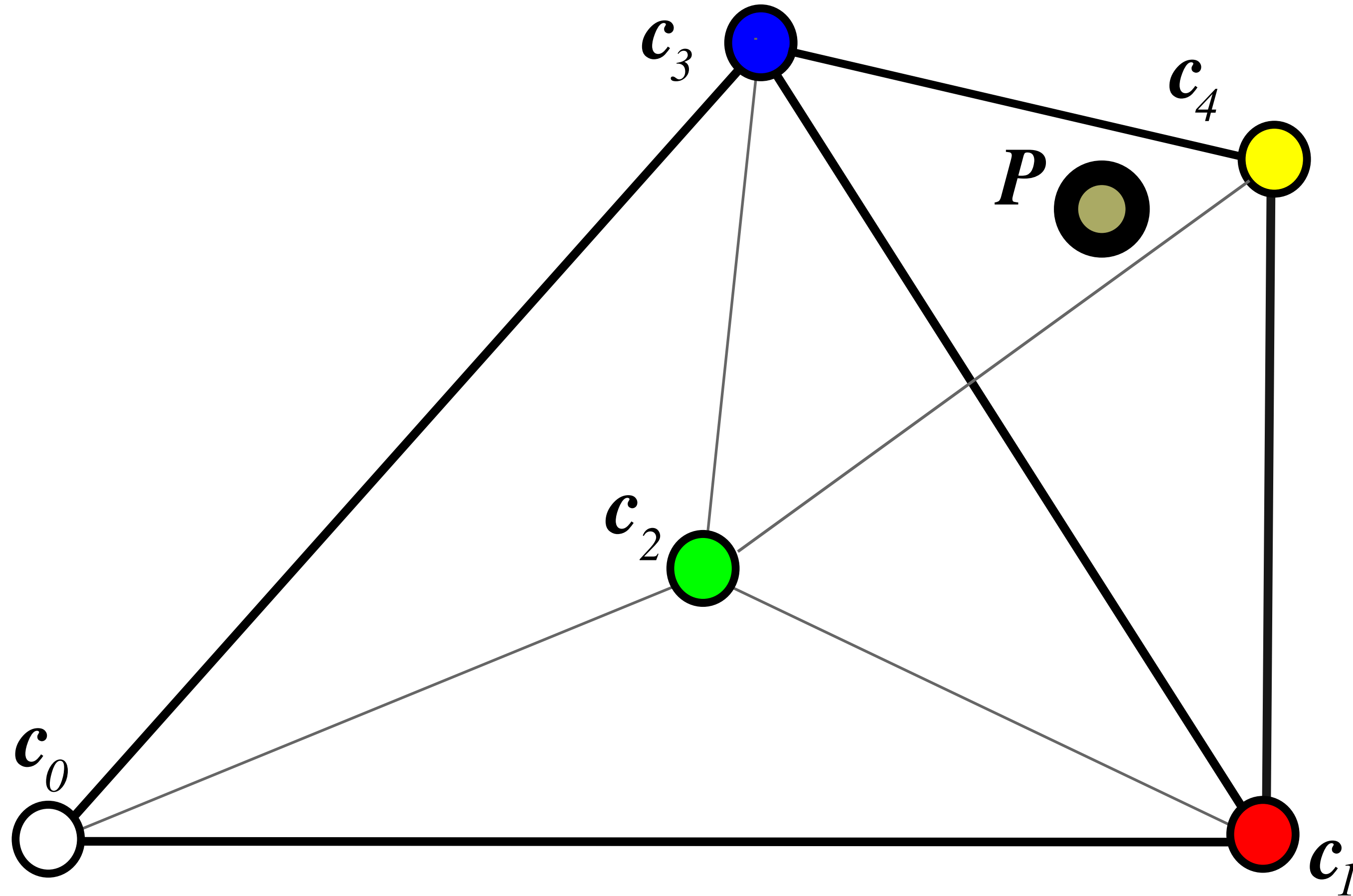
Path 1



# Color Compositing Path

- After user chooses a layer order:  $C_0 \rightarrow C_1 \rightarrow C_2 \rightarrow C_3 \rightarrow C_4$
- Still have **infinite** paths from  $C_0$  to  $P$

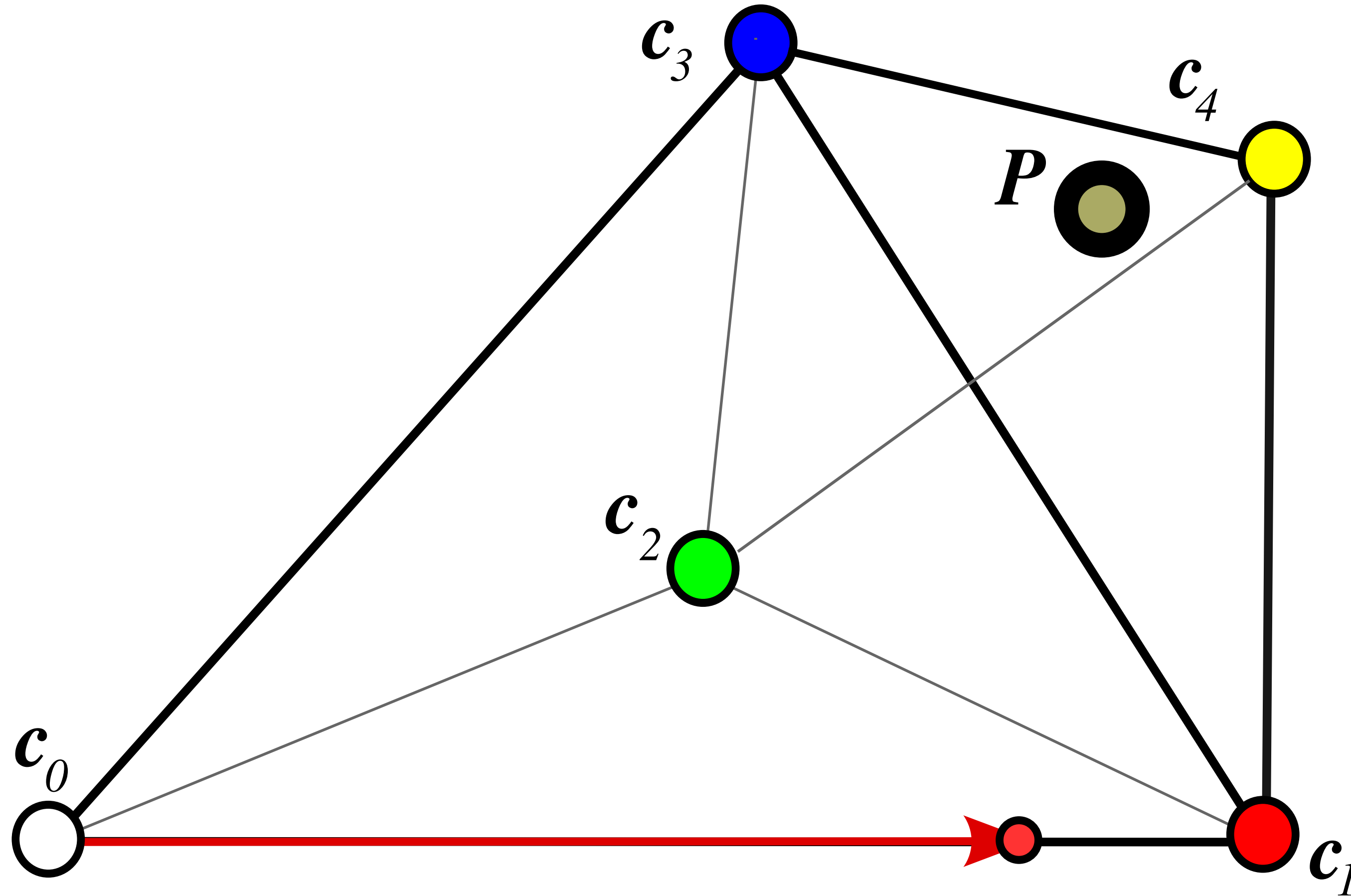
Path 2



# Color Compositing Path

- After user chooses a layer order:  $C_0 \rightarrow C_1 \rightarrow C_2 \rightarrow C_3 \rightarrow C_4$
- Still have **infinite** paths from  $C_0$  to  $P$

Path 2

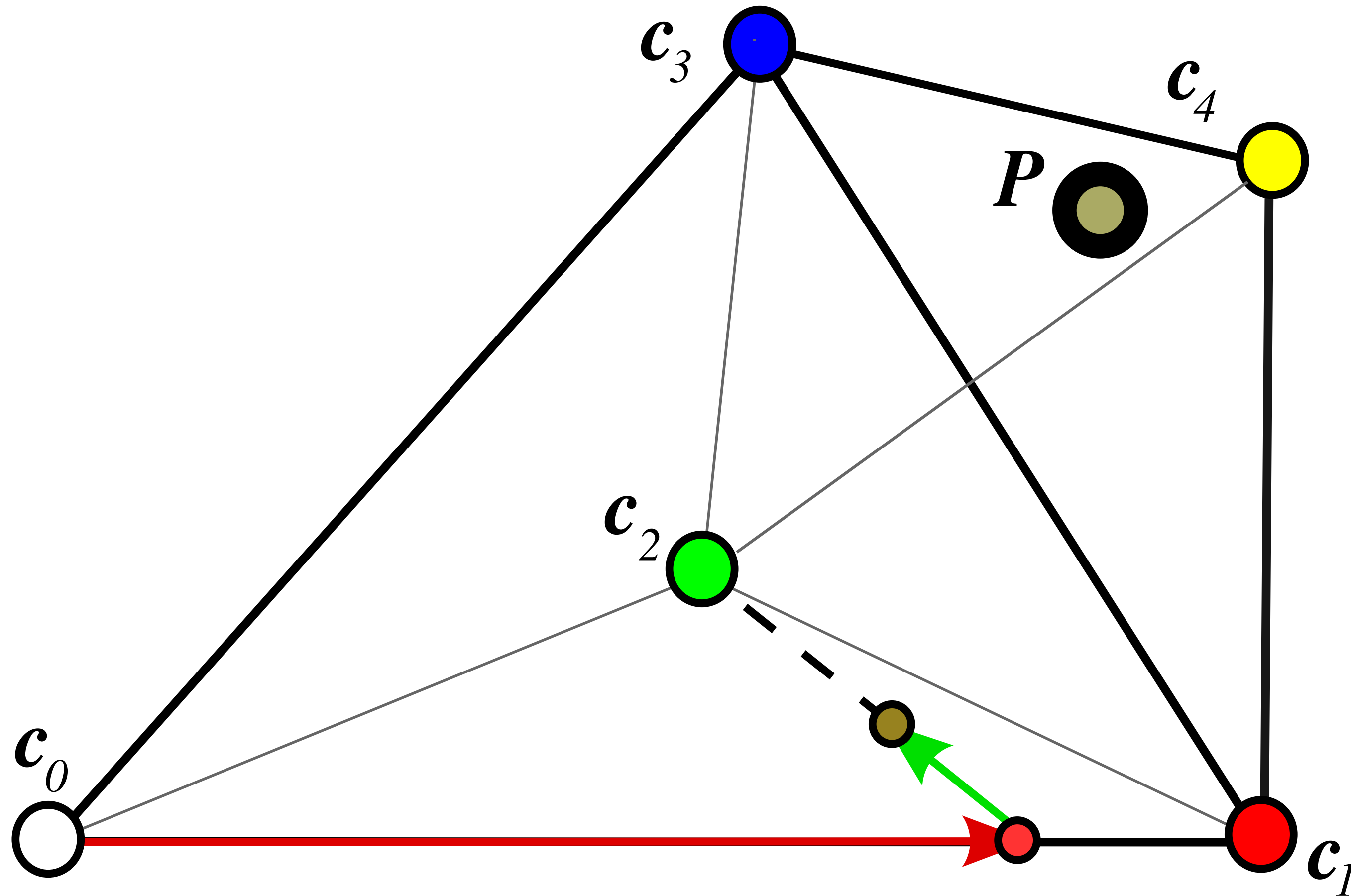




# Color Compositing Path

- After user chooses a layer order:  $C_0 \rightarrow C_1 \rightarrow C_2 \rightarrow C_3 \rightarrow C_4$
- Still have **infinite** paths from  $C_0$  to  $P$

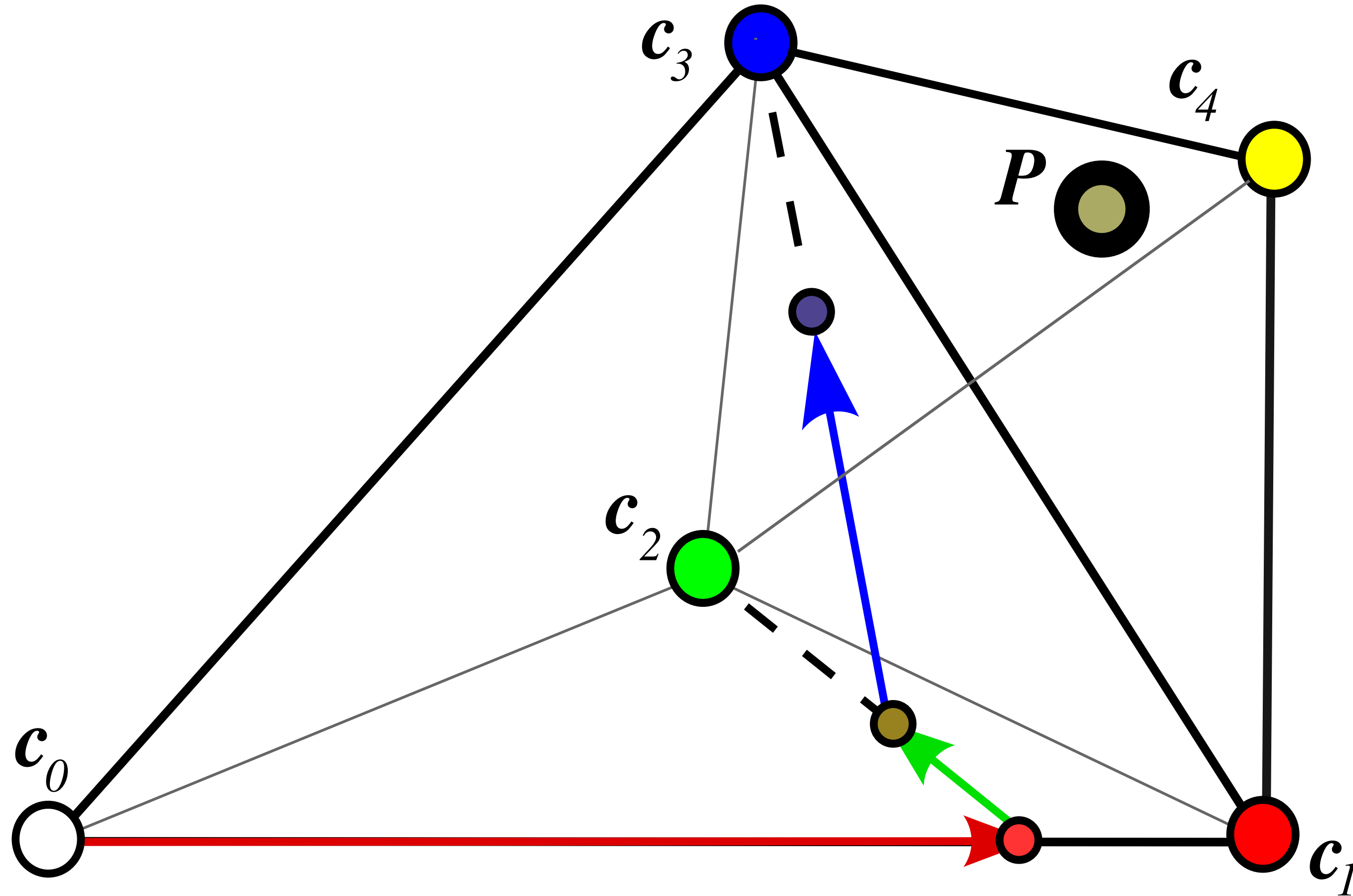
Path 2



# Color Compositing Path

- After user chooses a layer order:  $C_0 \rightarrow C_1 \rightarrow C_2 \rightarrow C_3 \rightarrow C_4$
- Still have **infinite** paths from  $C_0$  to  $P$

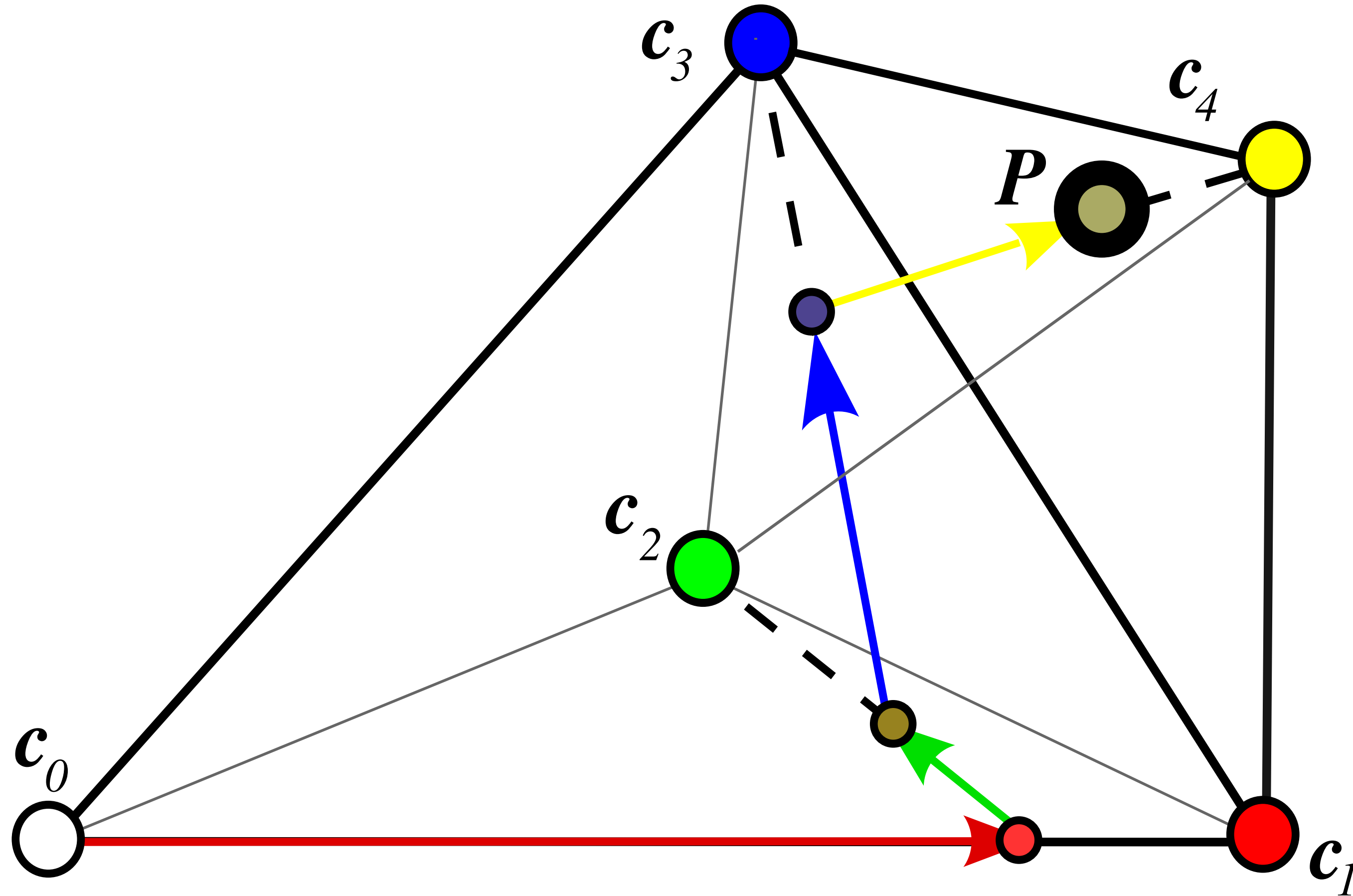
Path 2



# Color Compositing Path

- After user chooses a layer order:  $C_0 \rightarrow C_1 \rightarrow C_2 \rightarrow C_3 \rightarrow C_4$
- Still have **infinite** paths from  $C_0$  to  $P$

Path 2

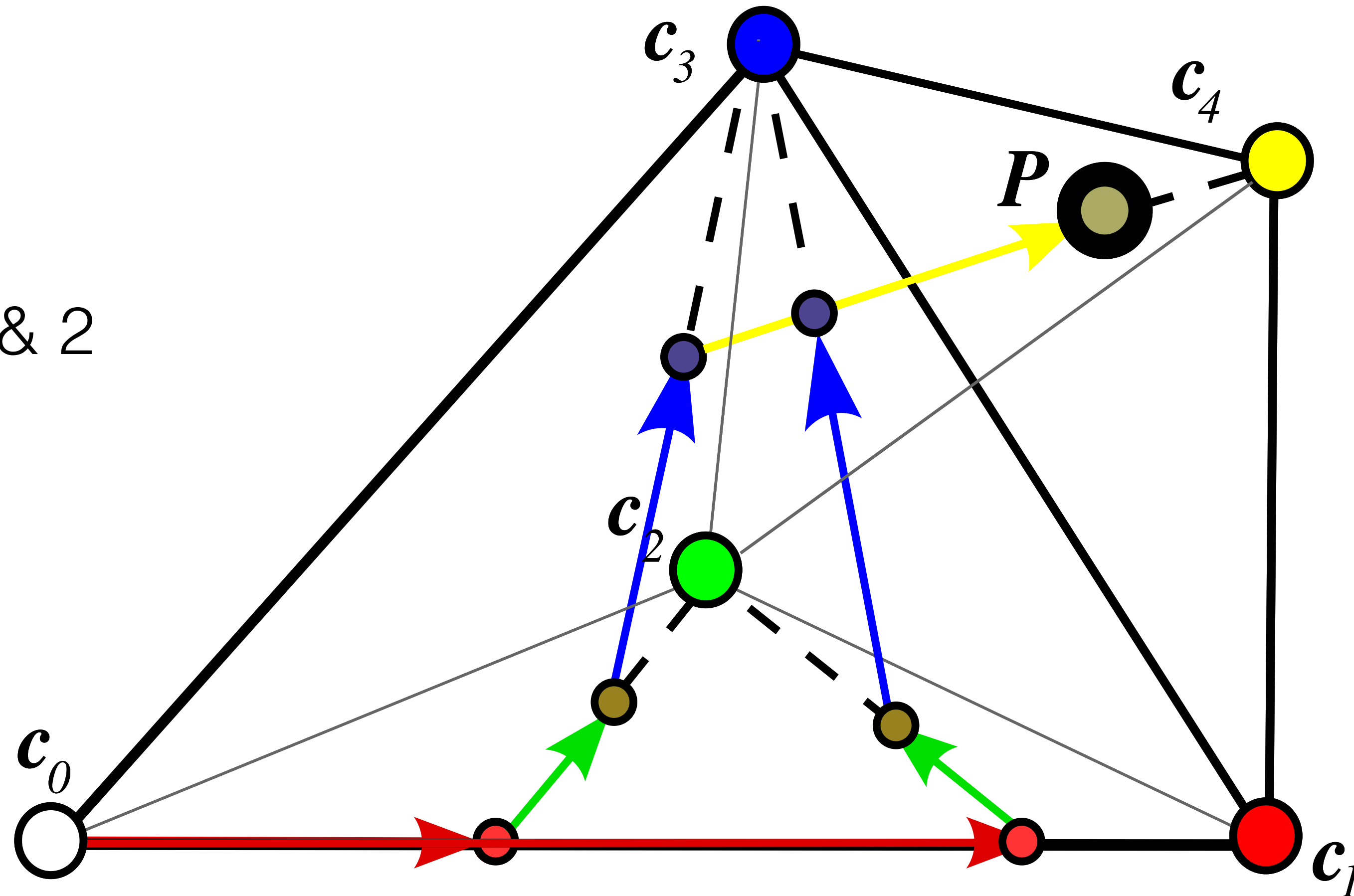




# Color Compositing Path

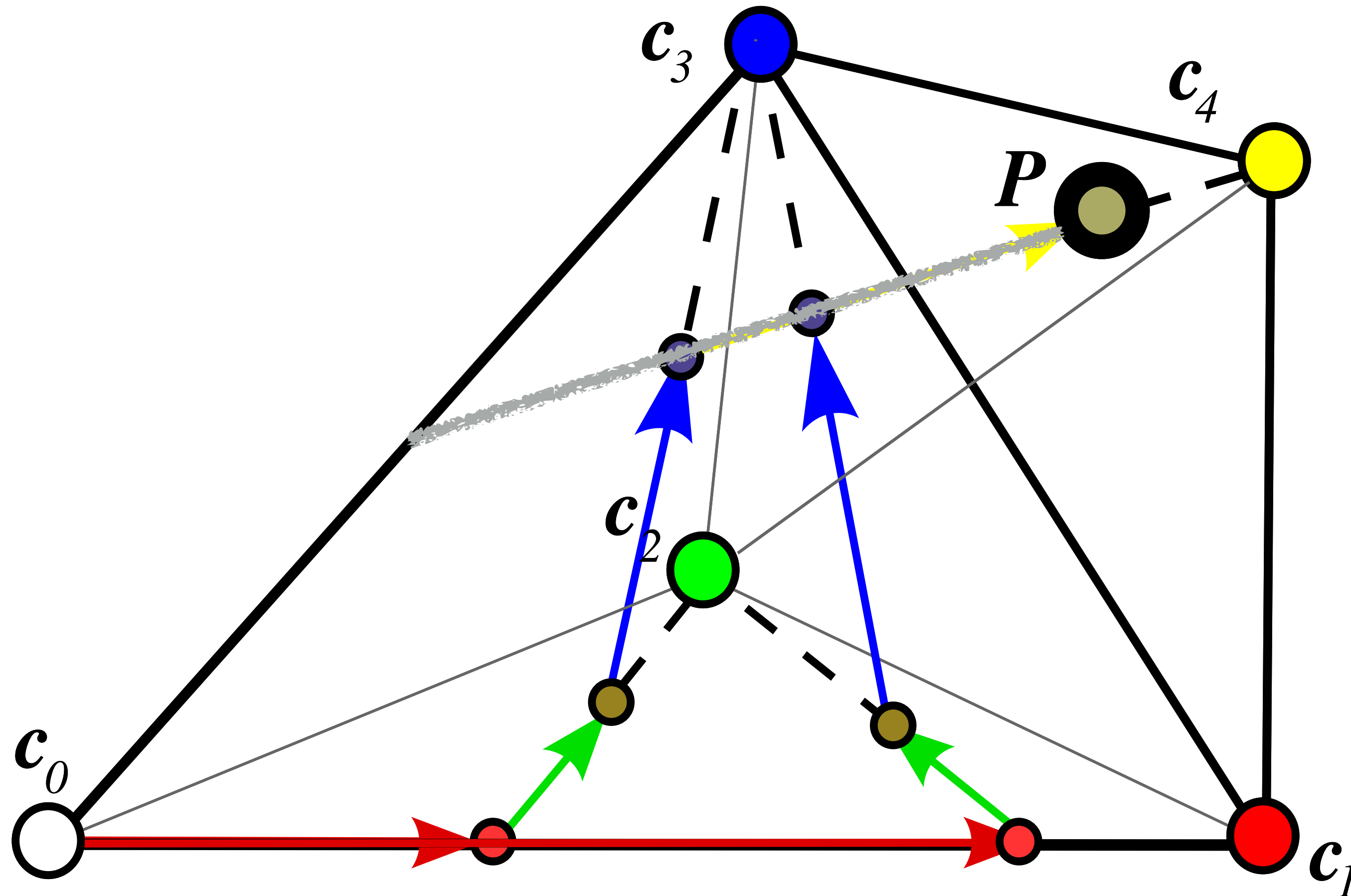
- After user chooses a layer order:  $C_0 \rightarrow C_1 \rightarrow C_2 \rightarrow C_3 \rightarrow C_4$
- Still have **infinite** paths from  $C_0$  to  $P$

Paths 1 & 2



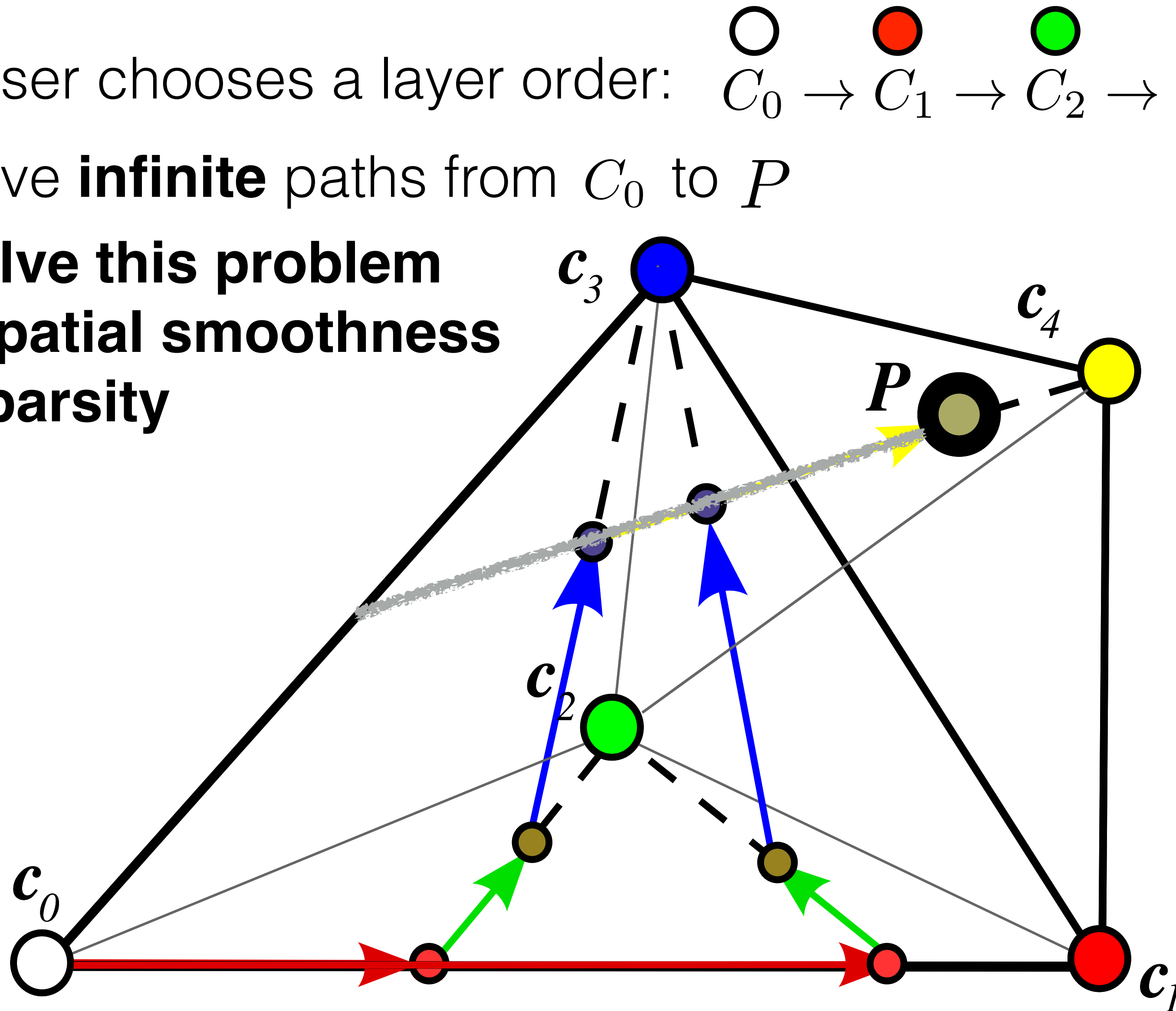
# Color Compositing Path

- After user chooses a layer order:  $C_0 \rightarrow C_1 \rightarrow C_2 \rightarrow C_3 \rightarrow C_4$
- Still have **infinite** paths from  $C_0$  to  $P$



# Color Compositing Path

- After user chooses a layer order:  $C_0 \rightarrow C_1 \rightarrow C_2 \rightarrow C_3 \rightarrow C_4$
- Still have **infinite** paths from  $C_0$  to  $P$
- **We solve this problem with spatial smoothness and sparsity**





# Layer Opacity Optimization

- Both **palette** and **palette order** are fixed now.
- We solve for layers' opacity values (**find a unique compositing path**)

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$$\| \text{original} - \text{reconstructed image} \|^2 \text{ (**polynomial**)}$$



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$$\begin{aligned} & \| \text{original} - \text{reconstructed image} \|^2 \text{ (**polynomial**)} \\ & + \\ & \text{Per pixel opacity sparsity } \sum -(1 - \alpha_i)^2 \end{aligned}$$

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- We solve for layers' opacity values (**find a unique compositing path**)
- We minimize an energy **E** =

$\| \text{original} - \text{reconstructed image} \|^2$  (**polynomial**)

+

Per pixel opacity sparsity  $\sum -(1 - \alpha_i)^2$

+

Opacity spatial smoothness (**Laplacian**)

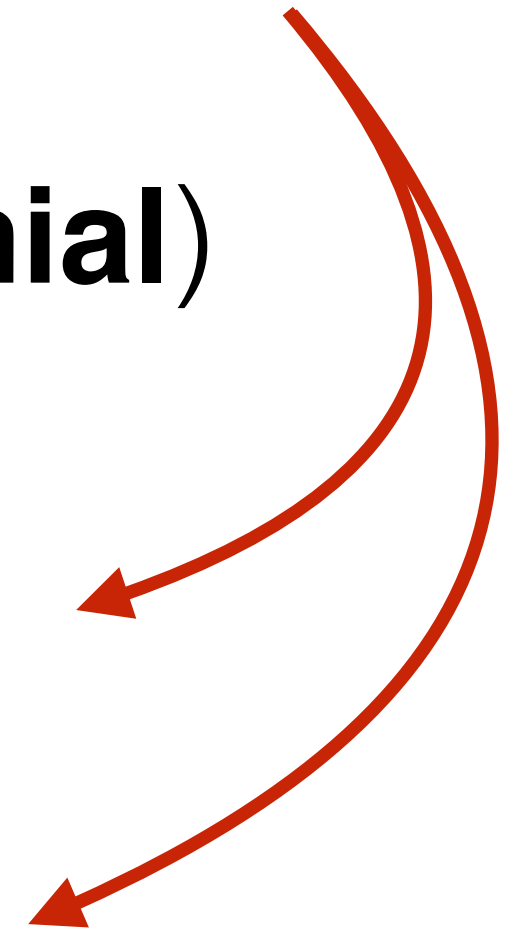
# Layer Opacity Optimization

- Both **palette** and **palette order** are fixed now.
- We solve for layers' opacity values (**find a unique compositing path**)
- We minimize an energy  $\mathbf{E} =$  **Shrink compositing paths' solution space**

$\| \text{original} - \text{reconstructed image} \|^2$  (**polynomial**)

+  
Per pixel opacity sparsity  $\sum -(1 - \alpha_i)^2$

+  
Opacity spatial smoothness (**Laplacian**)



# Layer Opacity Optimization

- Both **palette** and **palette order** are fixed now.
- We solve for layers' opacity values (**find a unique compositing path**)
- We minimize an energy **E** = **Shrink compositing paths' solution space**

$\| \text{original} - \text{reconstructed image} \|^2$  (**polynomial**)

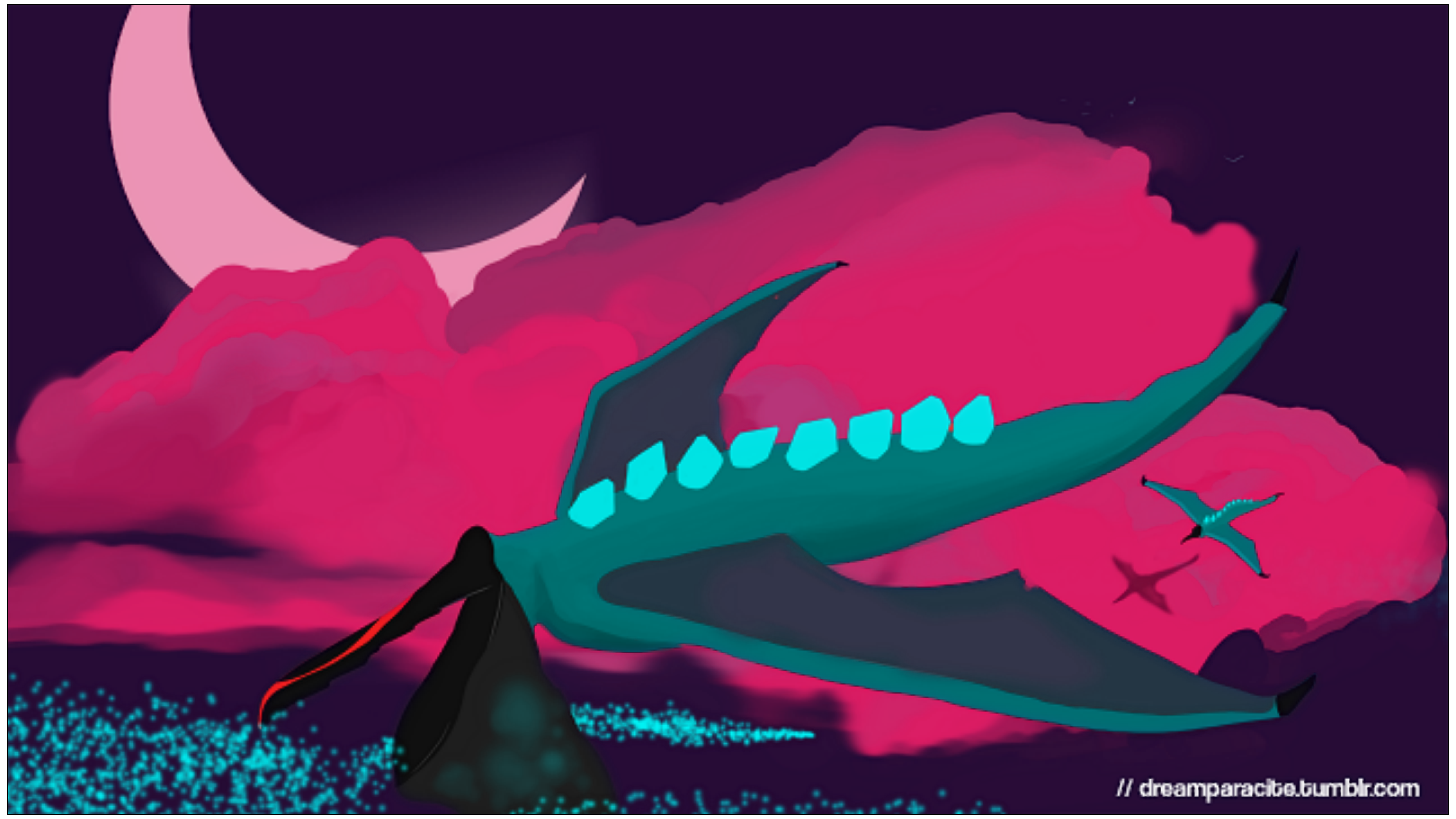
+  
Per pixel opacity sparsity  $\sum -(1 - \alpha_i)^2$

+  
Opacity spatial smoothness (**Laplacian**)

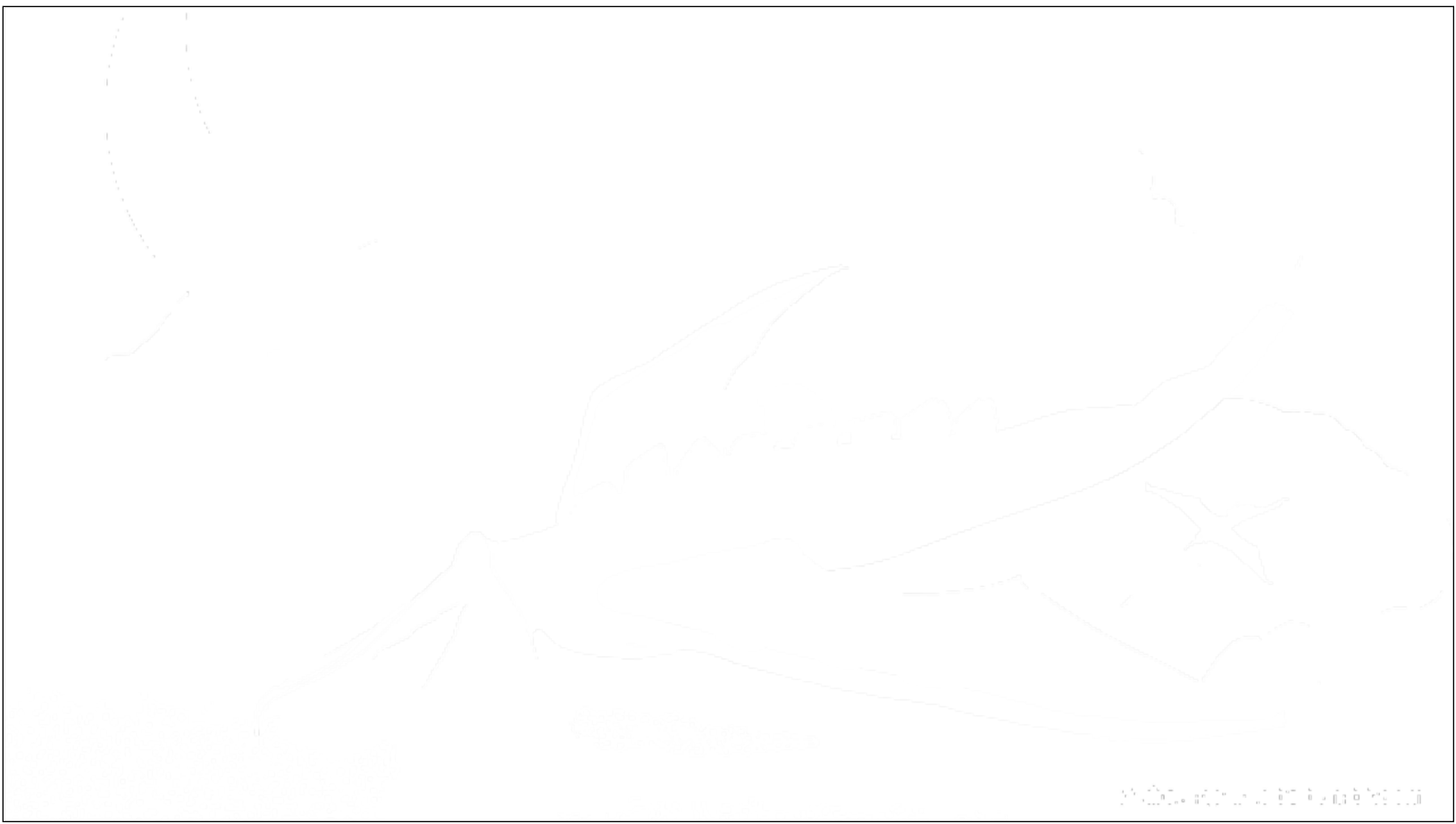
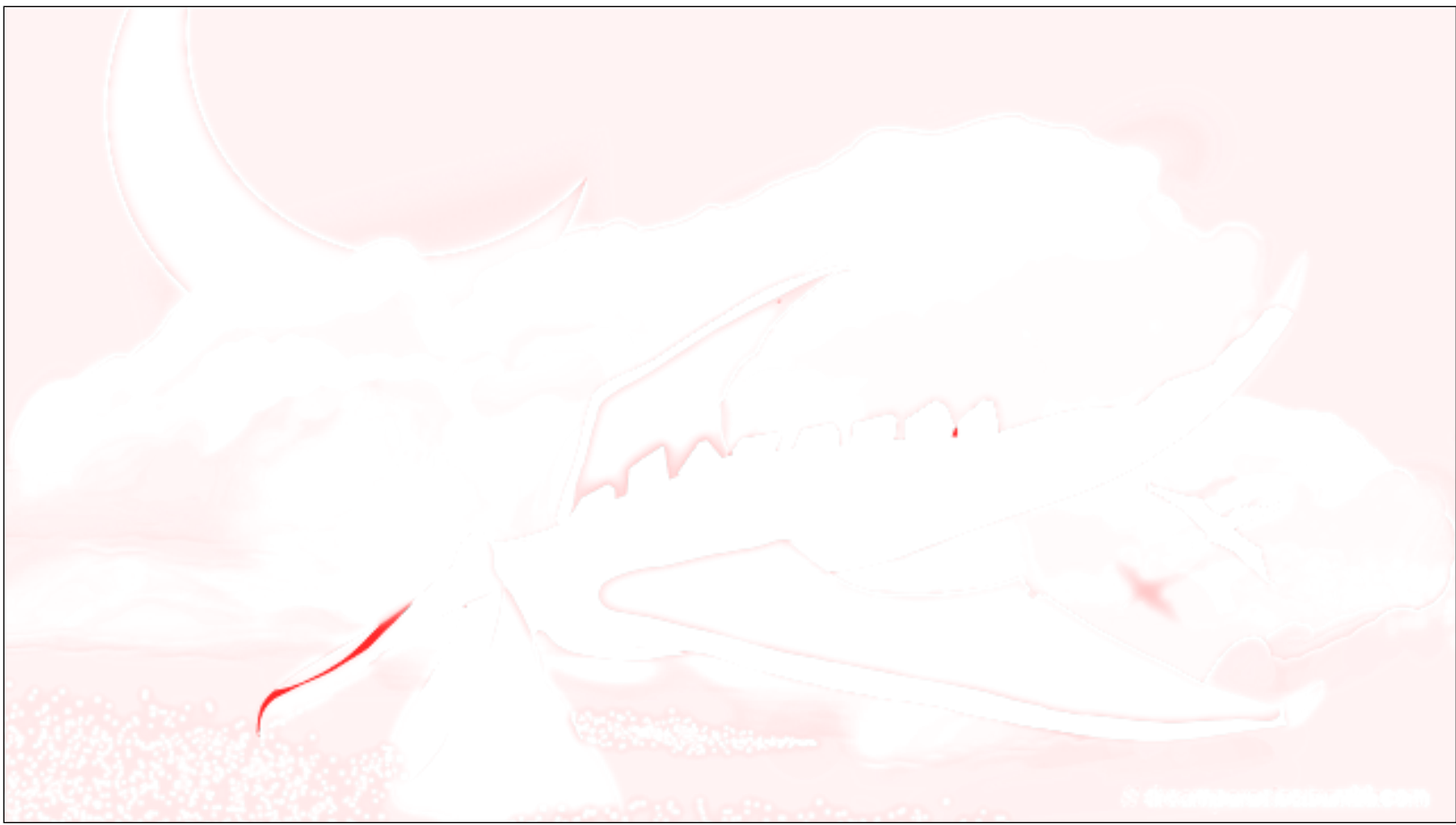
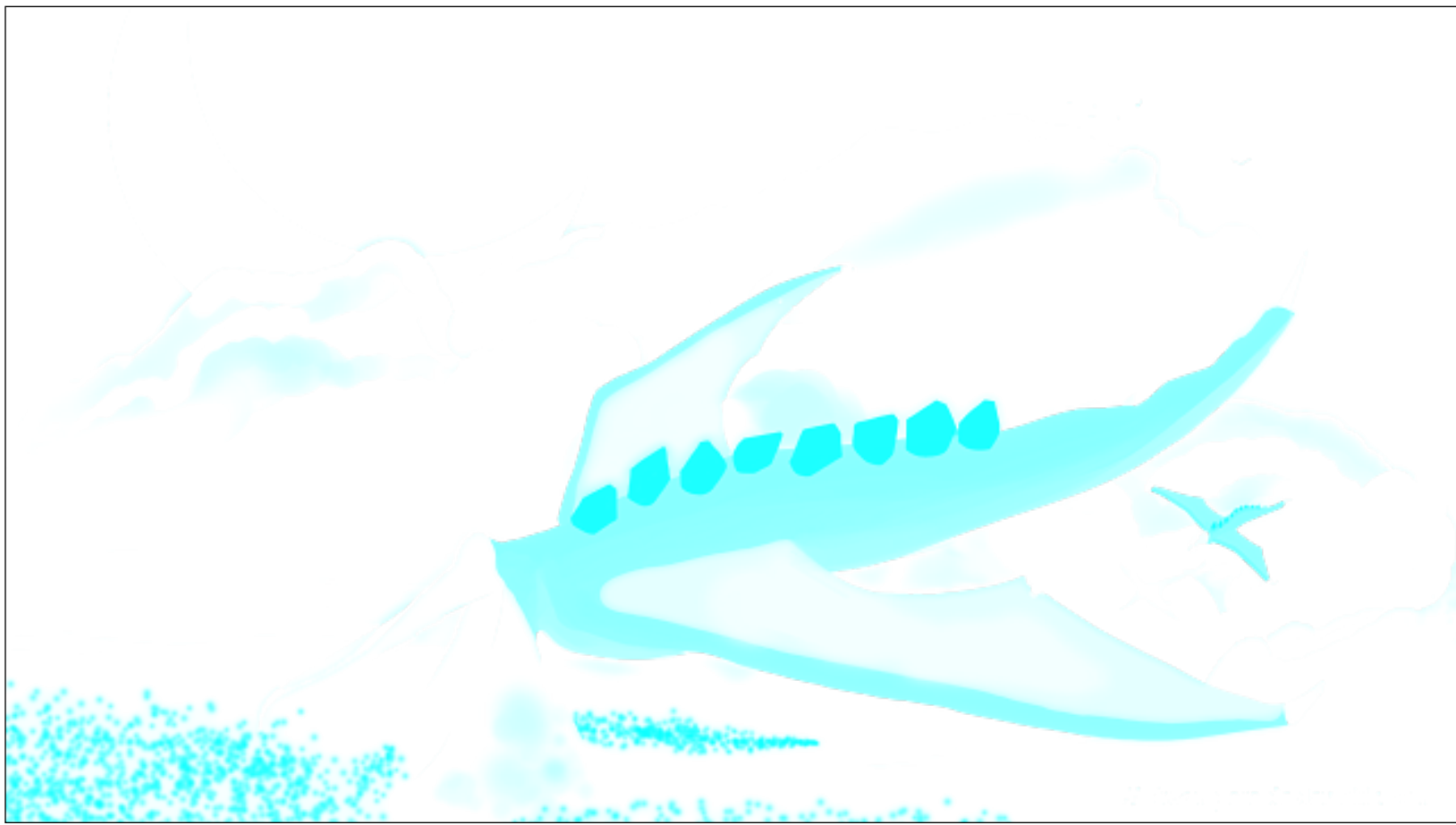
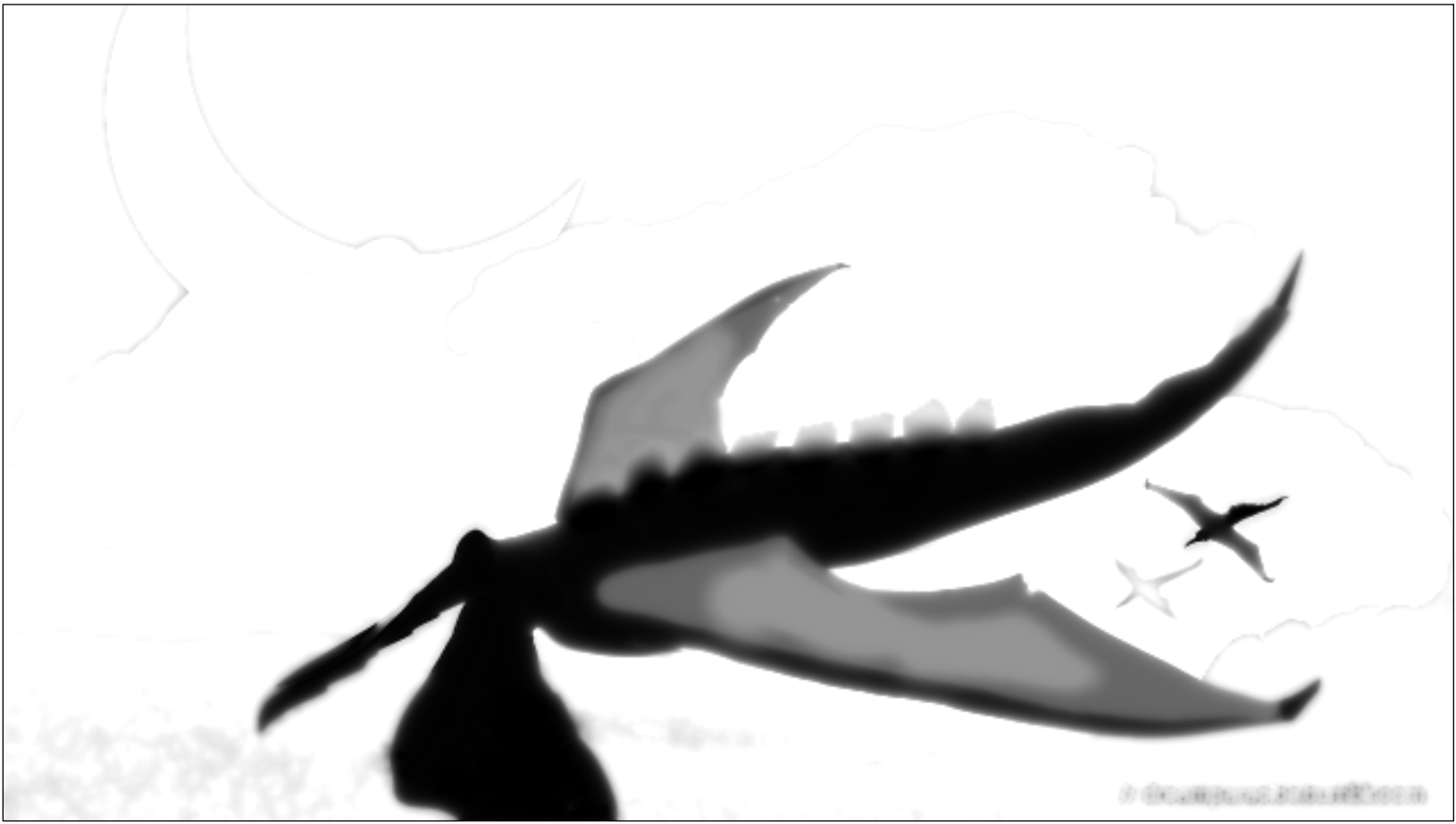
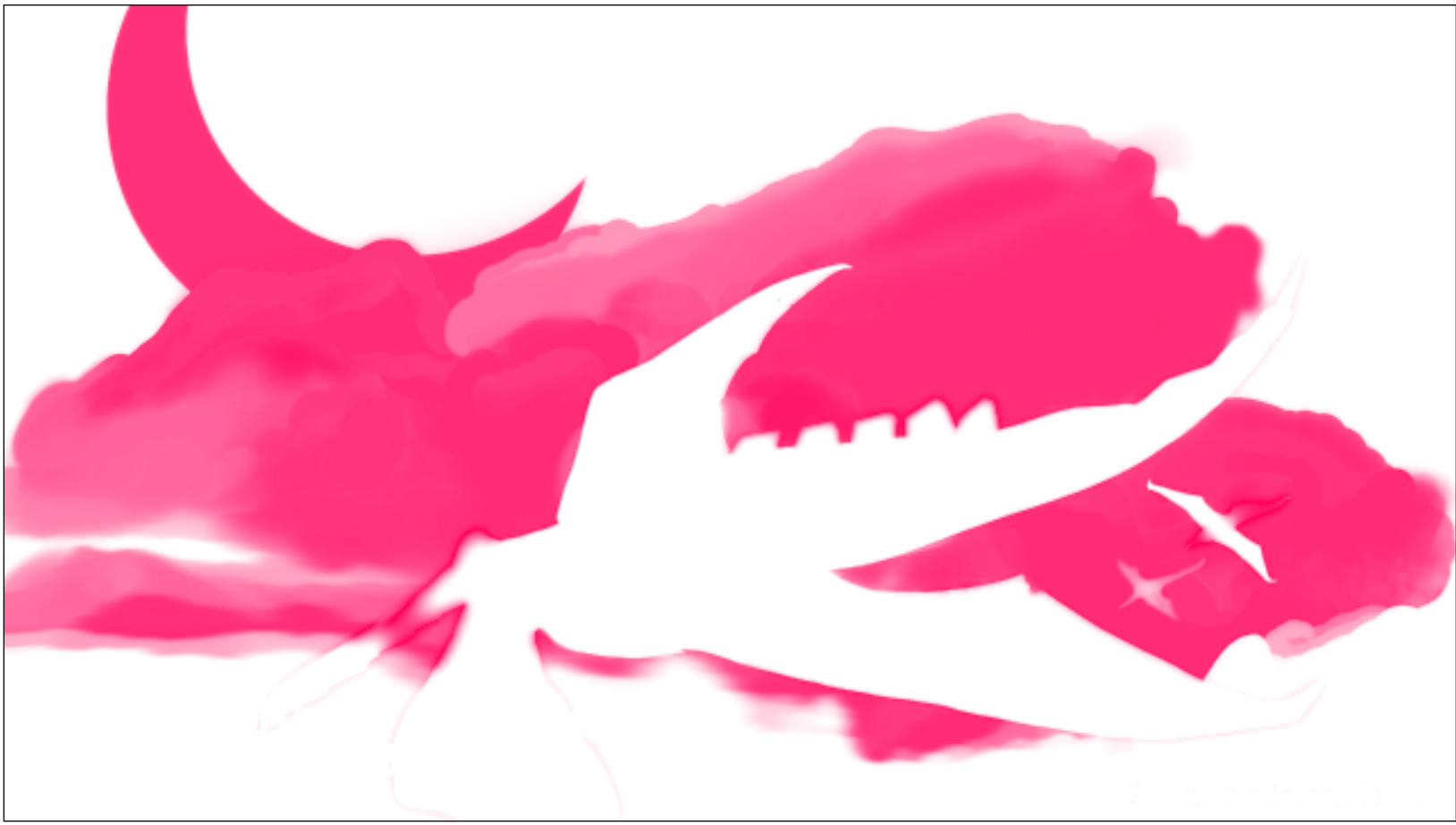
We also have a closed form expression for an “**As-Sparse-As-Possible**” solution if you don't care about spatial smoothness. See our paper for details.

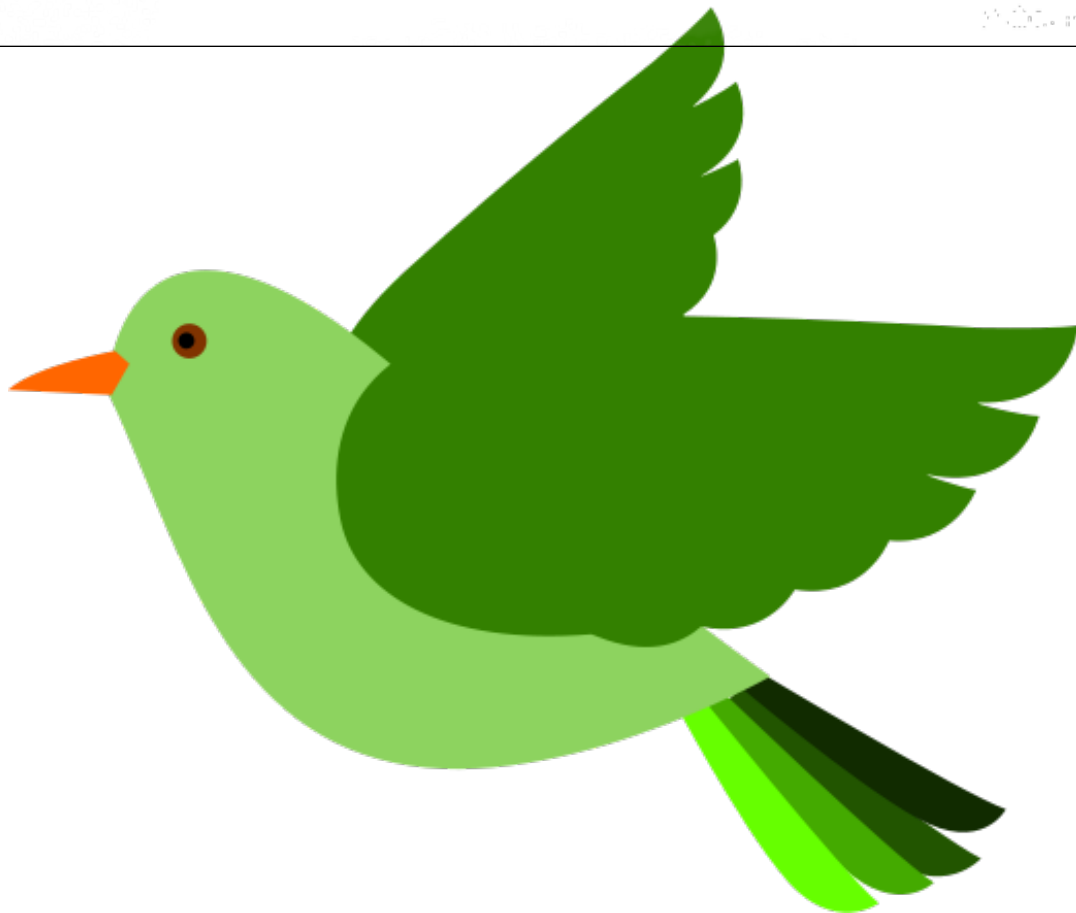
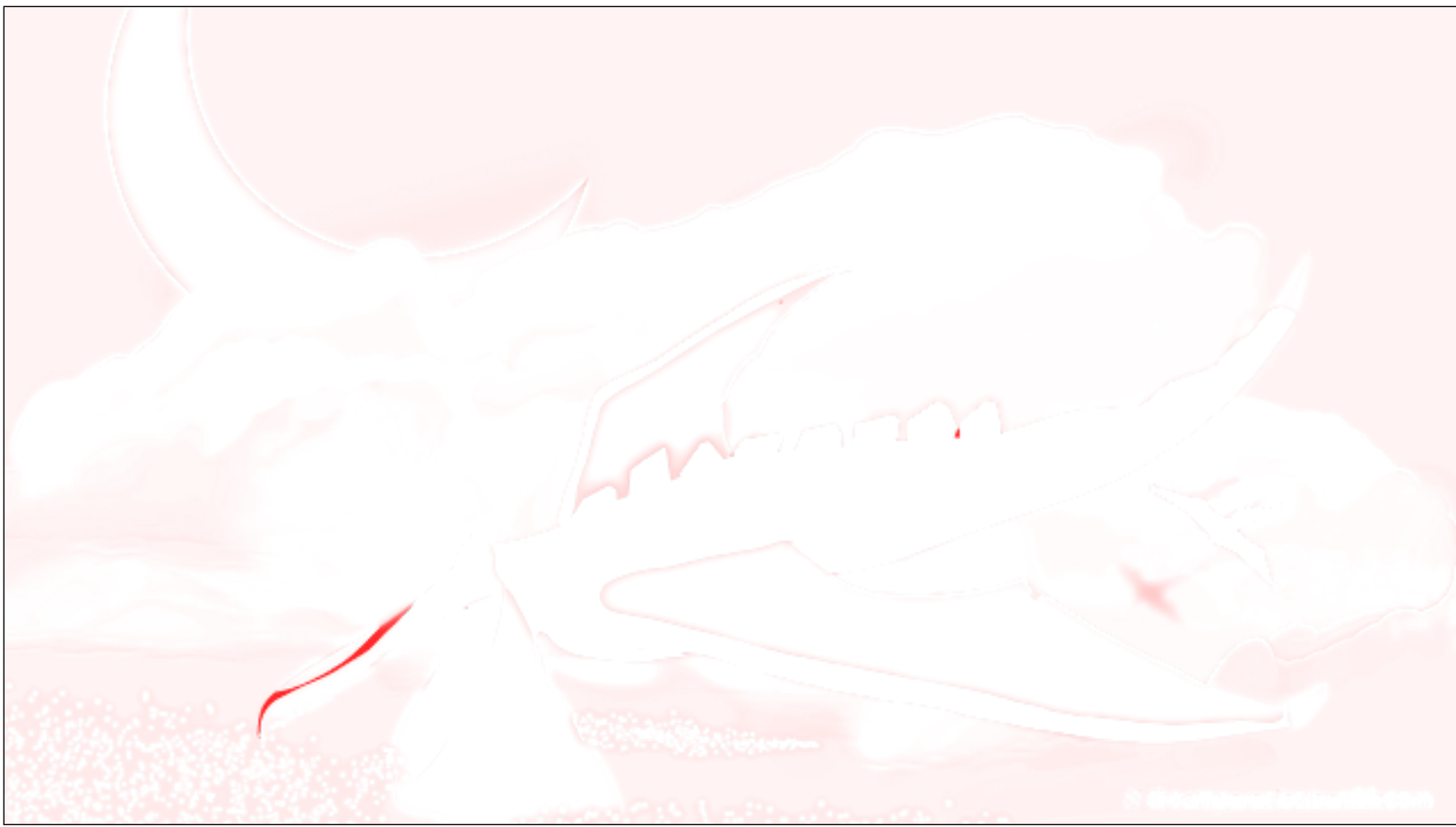
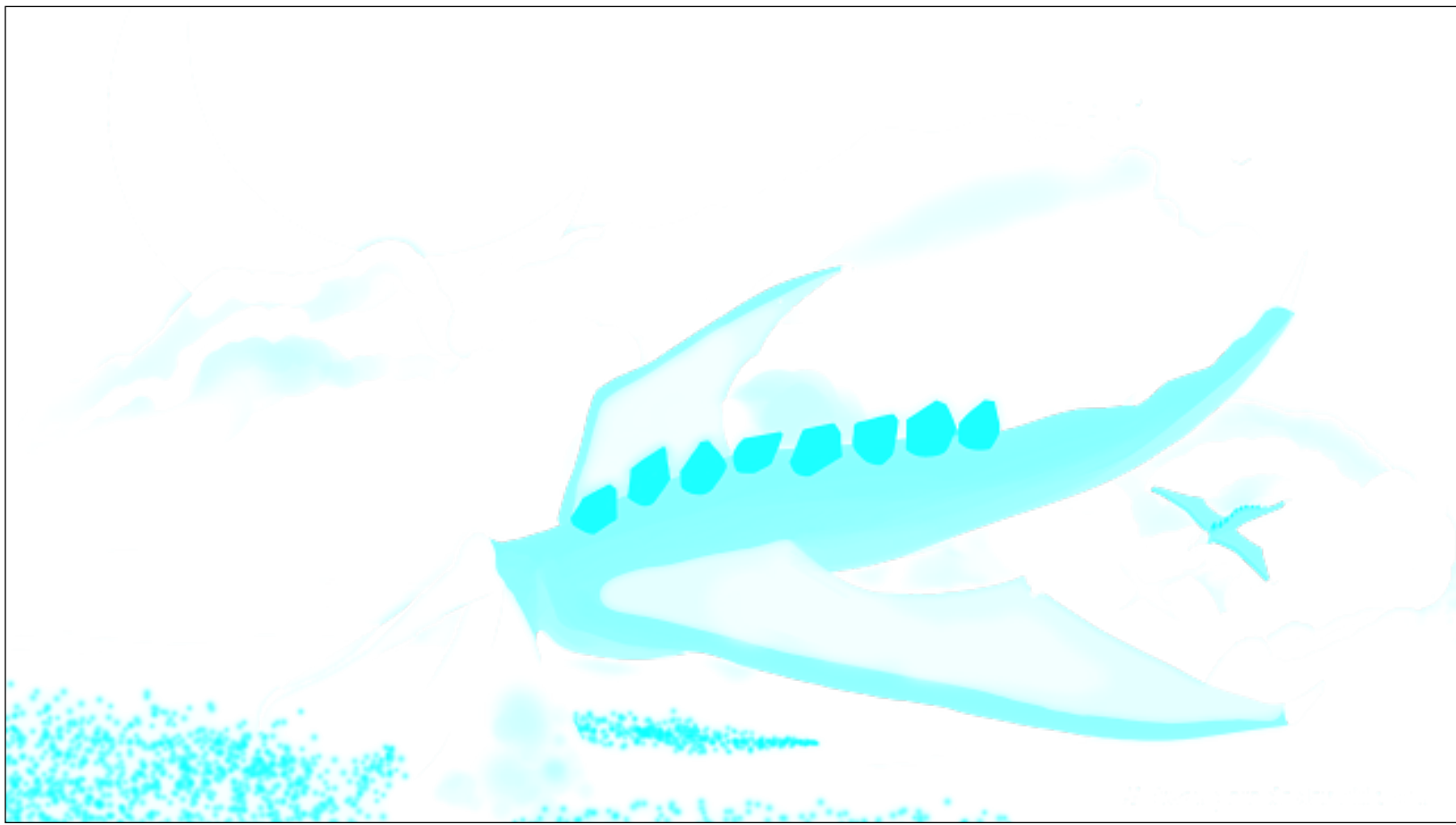
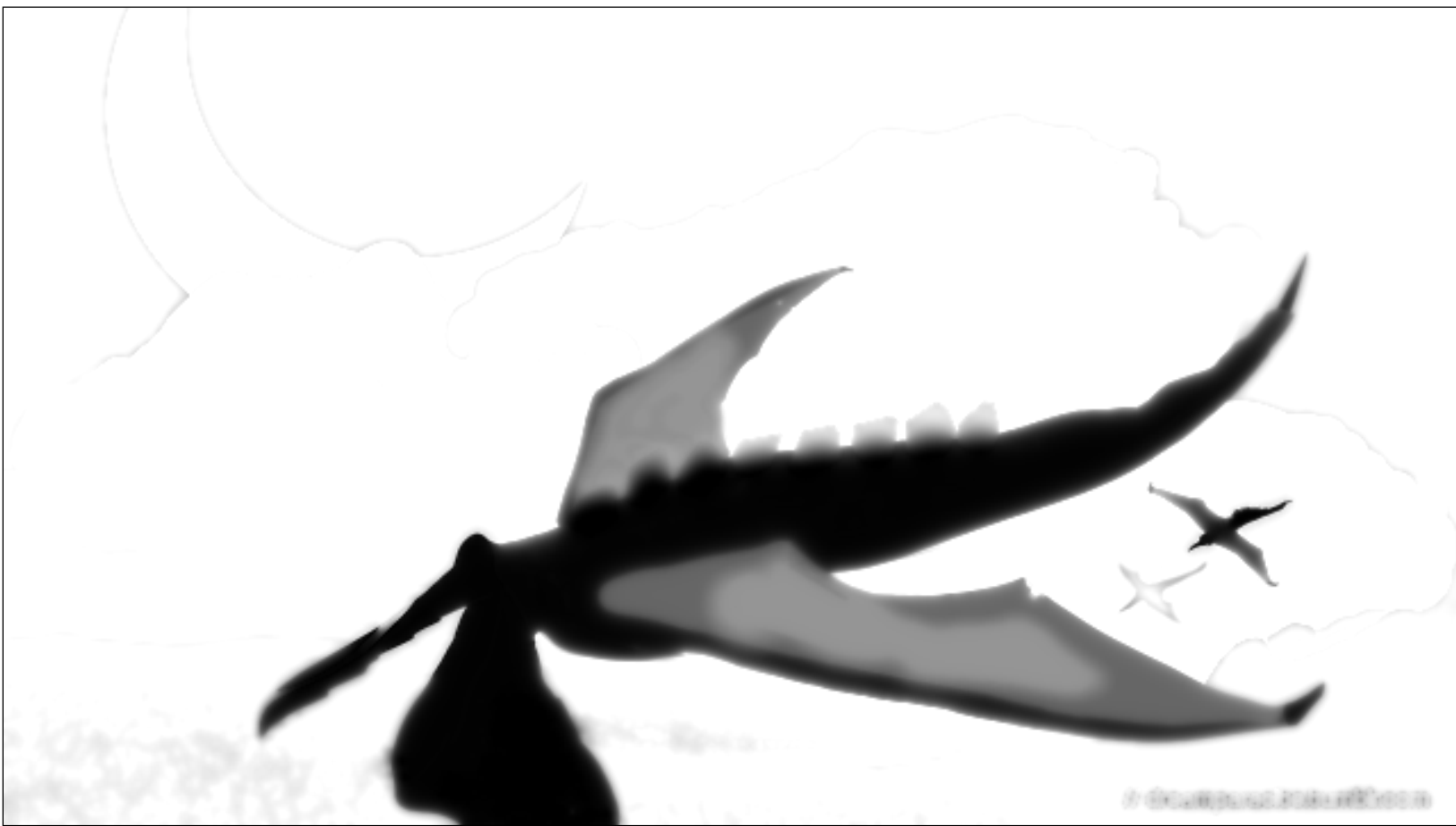


# Results





















# Local Recoloring

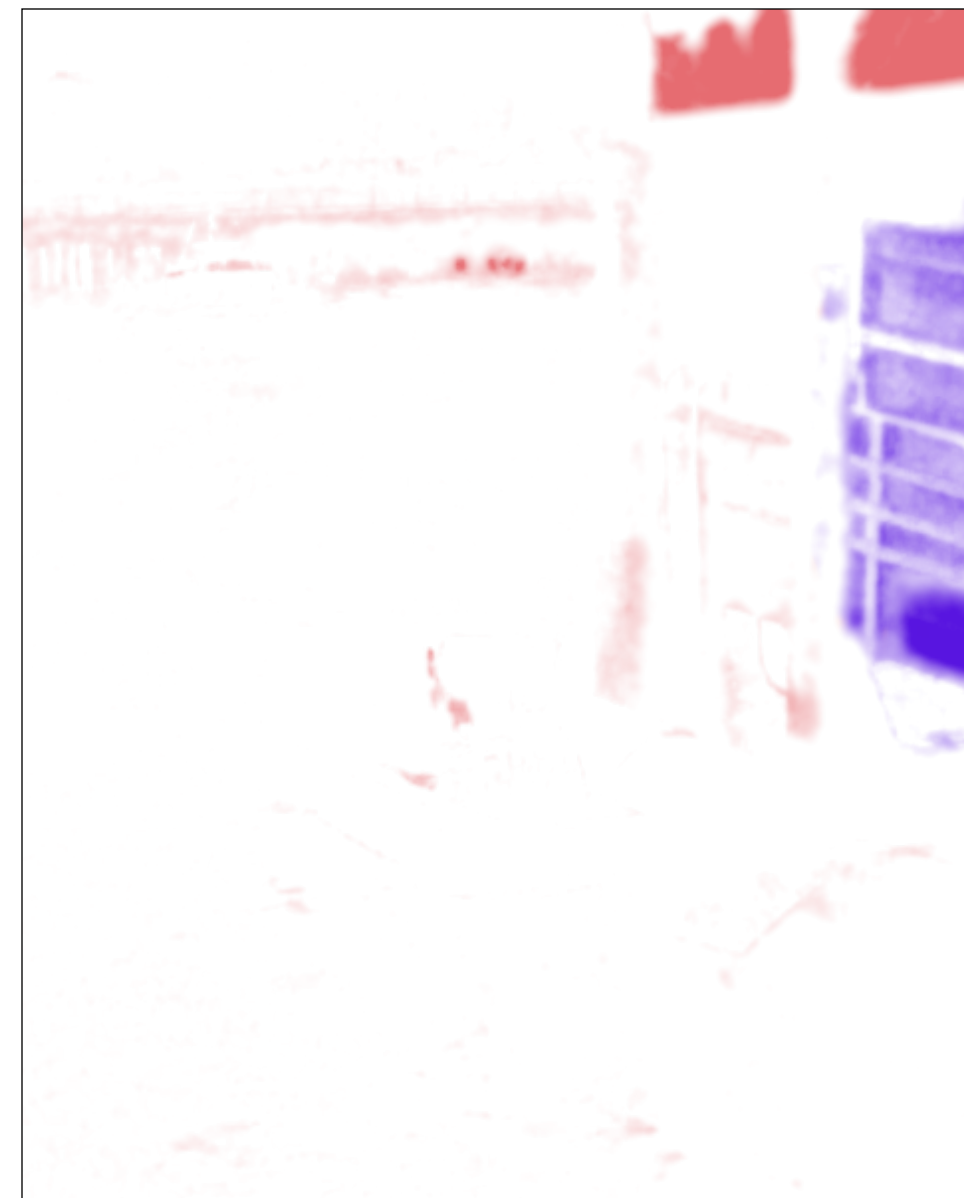
Original





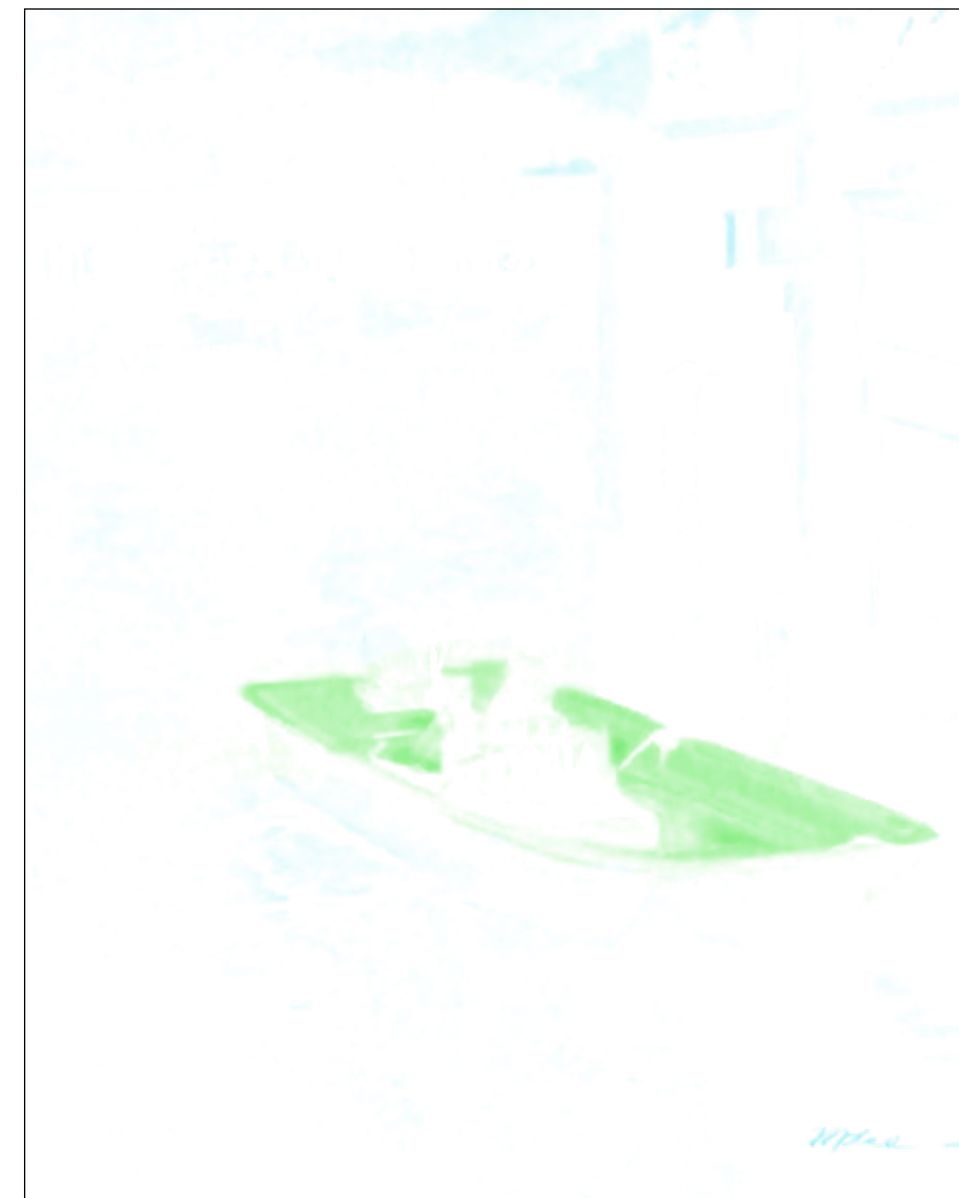
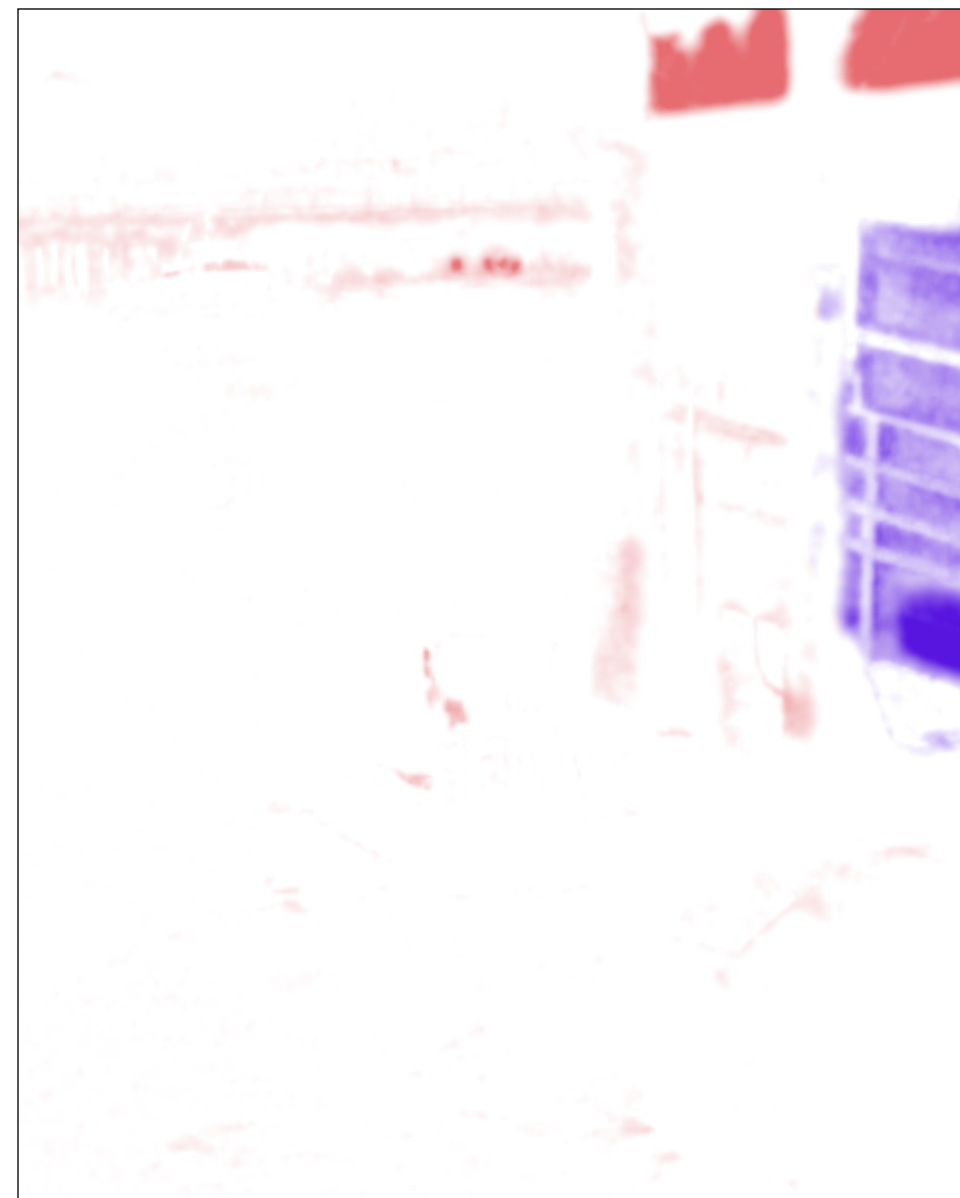
# Local Recoloring

Original



# Local Recoloring

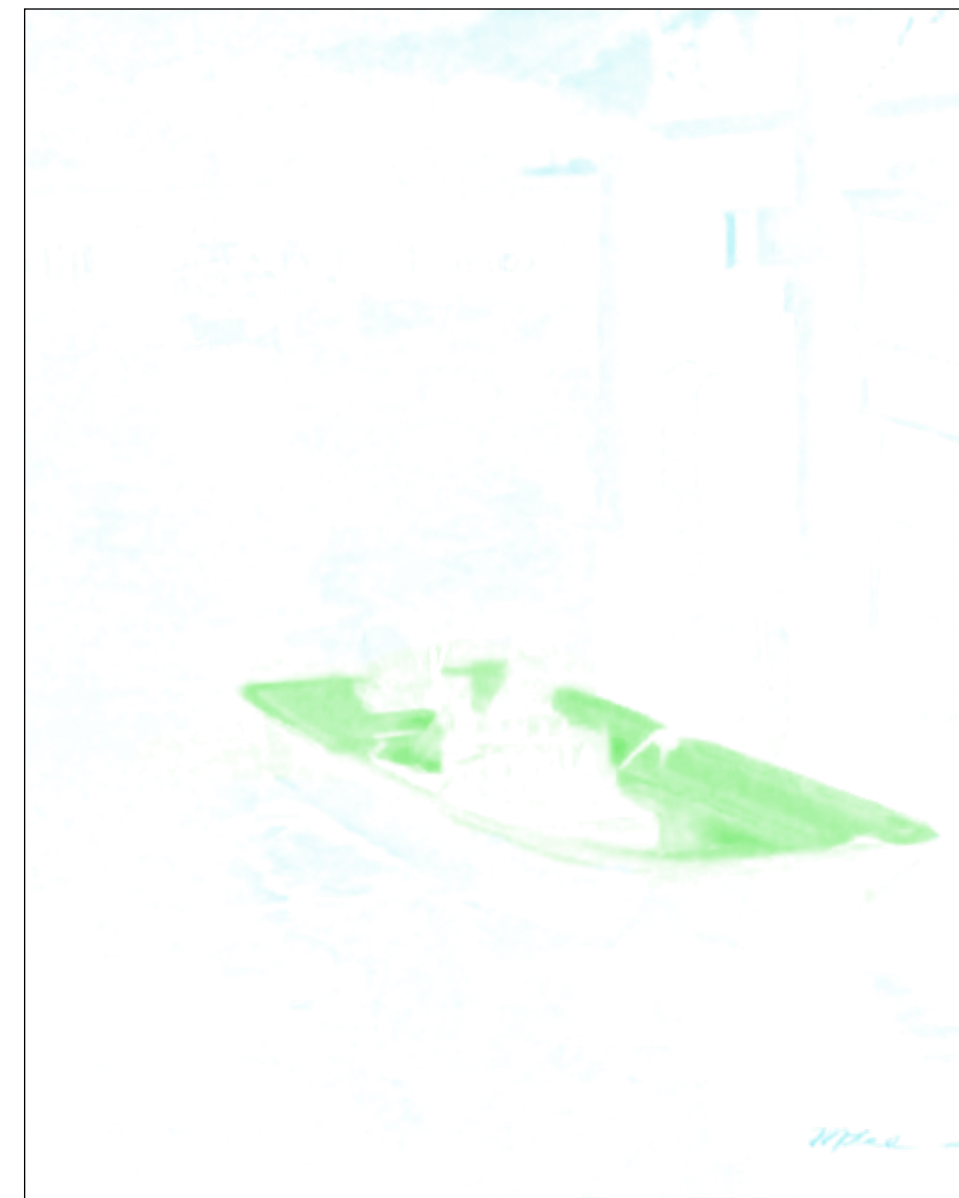
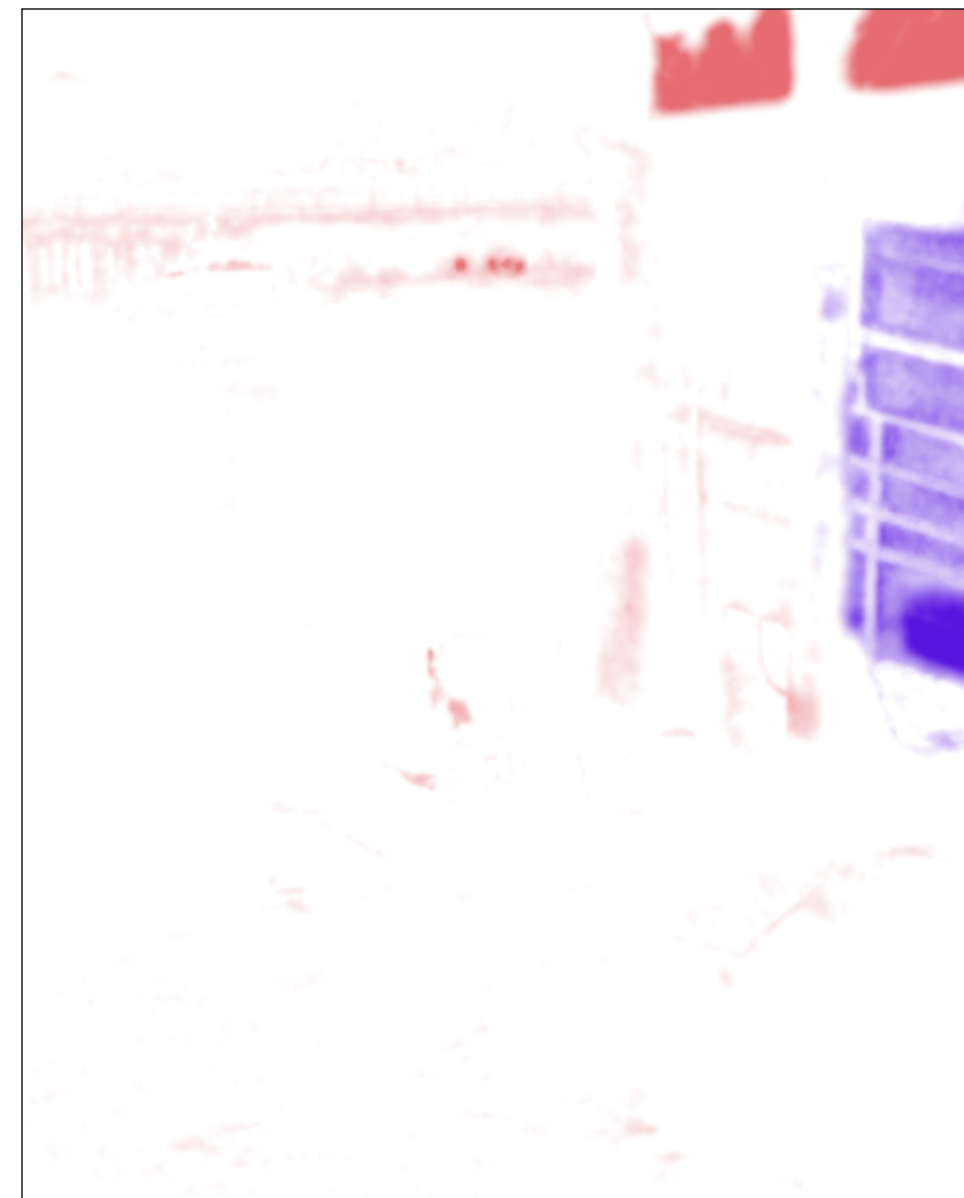
Original



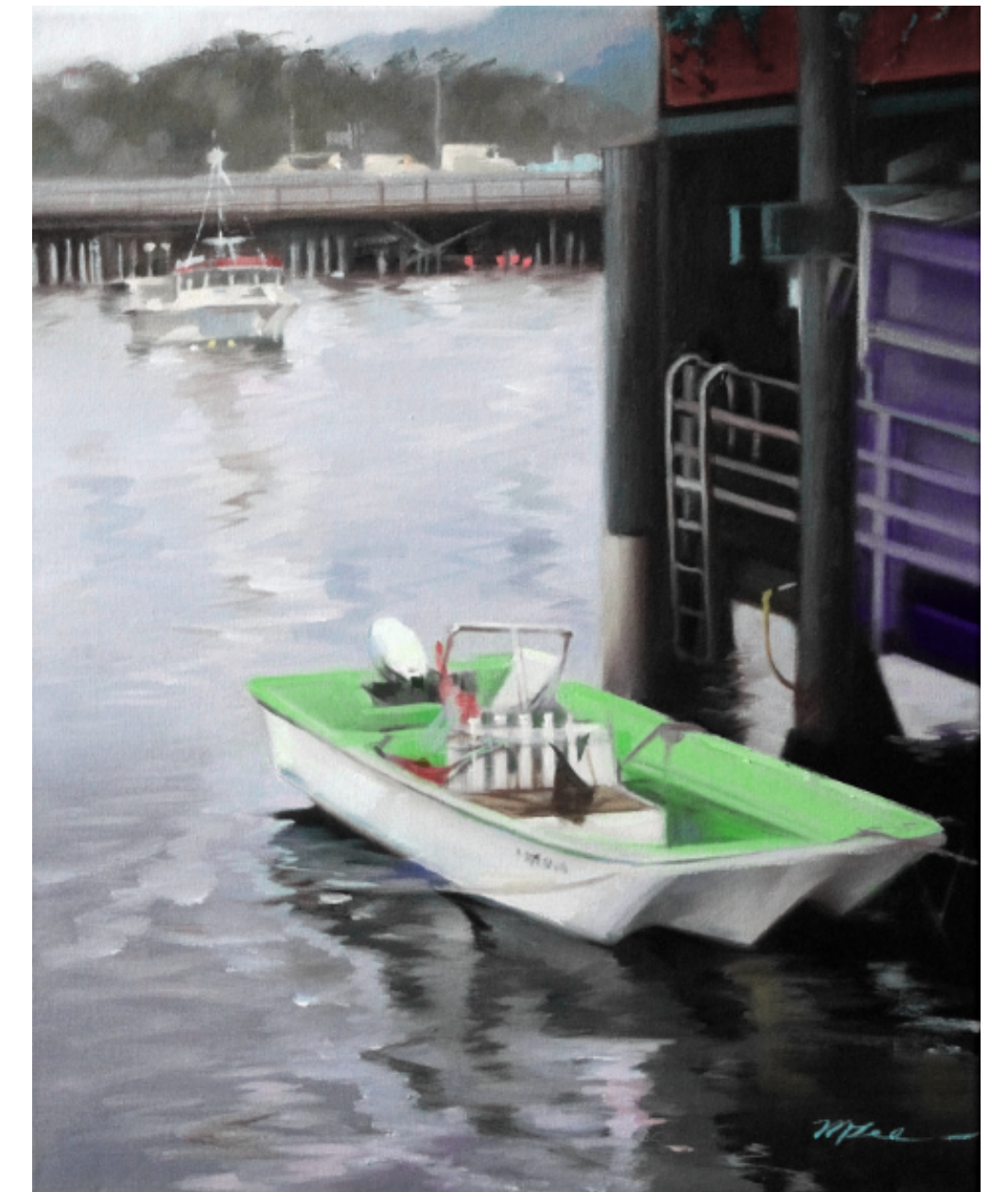


# Local Recoloring

Original



Modified





# Generalized Barycentric Coordinates

$$p = \sum w_i C_i$$

# Generalized Barycentric Coordinates

$$p = \sum w_i c_i \quad \text{linear mixing weights}$$

# Generalized Barycentric Coordinates

$$p = \sum w_i c_i \quad \text{linear mixing weights}$$

$$p = c_n + \sum_{i=1}^n \left[ (c_{i-1} - c_i) \prod_{j=i}^n (1 - \alpha_j) \right]$$

## Alpha Compositing



# Generalized Barycentric Coordinates

$$p = \sum w_i c_i \quad \text{linear mixing weights}$$

$$p = c_n + \sum_{i=1}^n \left[ (c_{i-1} - c_i) \prod_{j=i}^n (1 - \alpha_j) \right] \quad \text{layer opacities}$$

## Alpha Compositing

# Generalized Barycentric Coordinates

$$p = \sum w_i c_i \quad \text{linear mixing weights}$$

Unique

$$p = c_n + \sum_{i=1}^n \left[ (c_{i-1} - c_i) \prod_{j=i}^n (1 - \alpha_j) \right] \quad \text{layer opacities}$$

## Alpha Compositing

# Generalized Barycentric Coordinates

$$p = \sum w_i c_i \quad \text{linear mixing weights}$$

Unique

Ambiguous

$$p = c_n + \sum_{i=1}^n \left[ (c_{i-1} - c_i) \prod_{j=i}^n (1 - \alpha_j) \right] \quad \text{layer opacities}$$

## Alpha Compositing

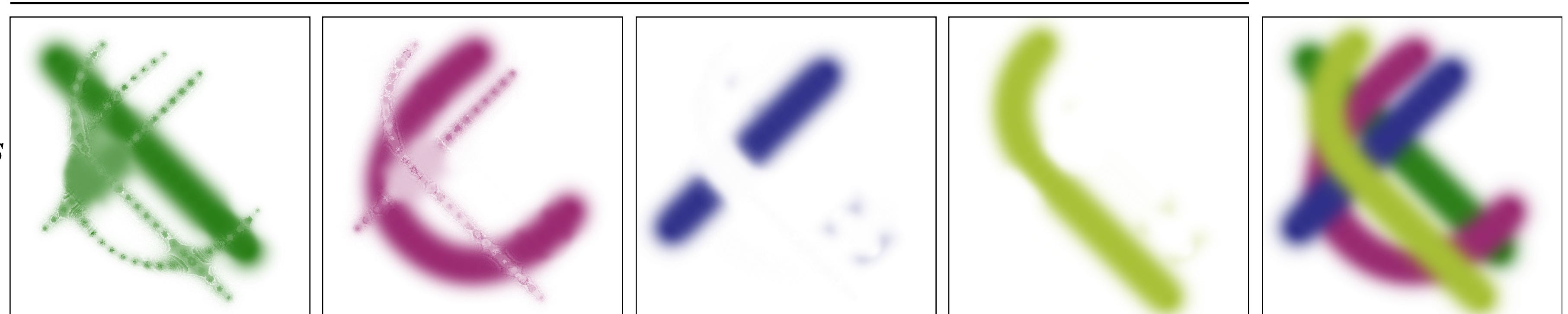


# Generalized Barycentric Coordinates

*layers*

*composite*

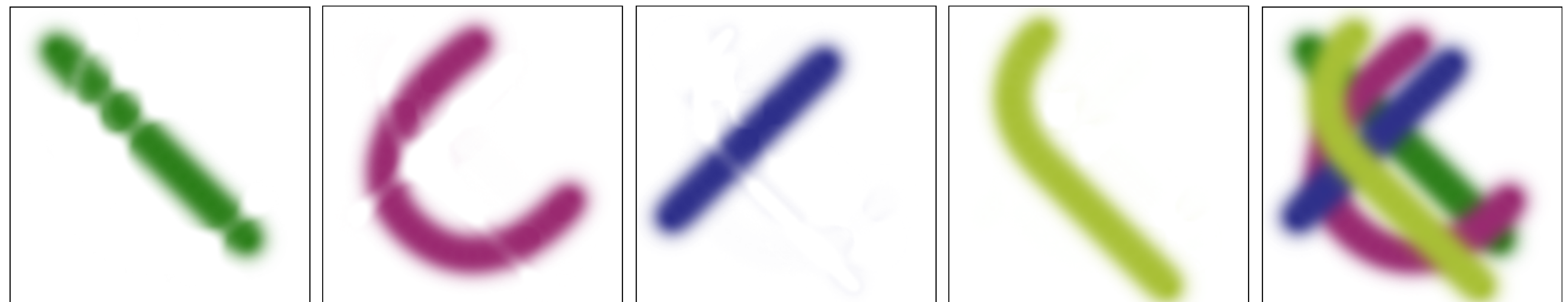
***MVC***  
***Mean Value Coordinates***  
*[Floater et al. 2005]*  
*[Ju et al. 2005]*



***LBC***  
***Local Bary. Coordinates***  
*[Zhang et al. 2014]*



***Ours***



# Demo

**Our layer opacities can also be uniquely converted into Generalized Barycentric Coordinates.**

**Actually, we now get additive mixing layers, which is layer order independent.**

**Global Recoloring**



Visualize the colors of an image as a 3D RGB point cloud.

[apple.png](#)



width: 500, height: 453  
total pixels: 226500  
unique pixels: 226500

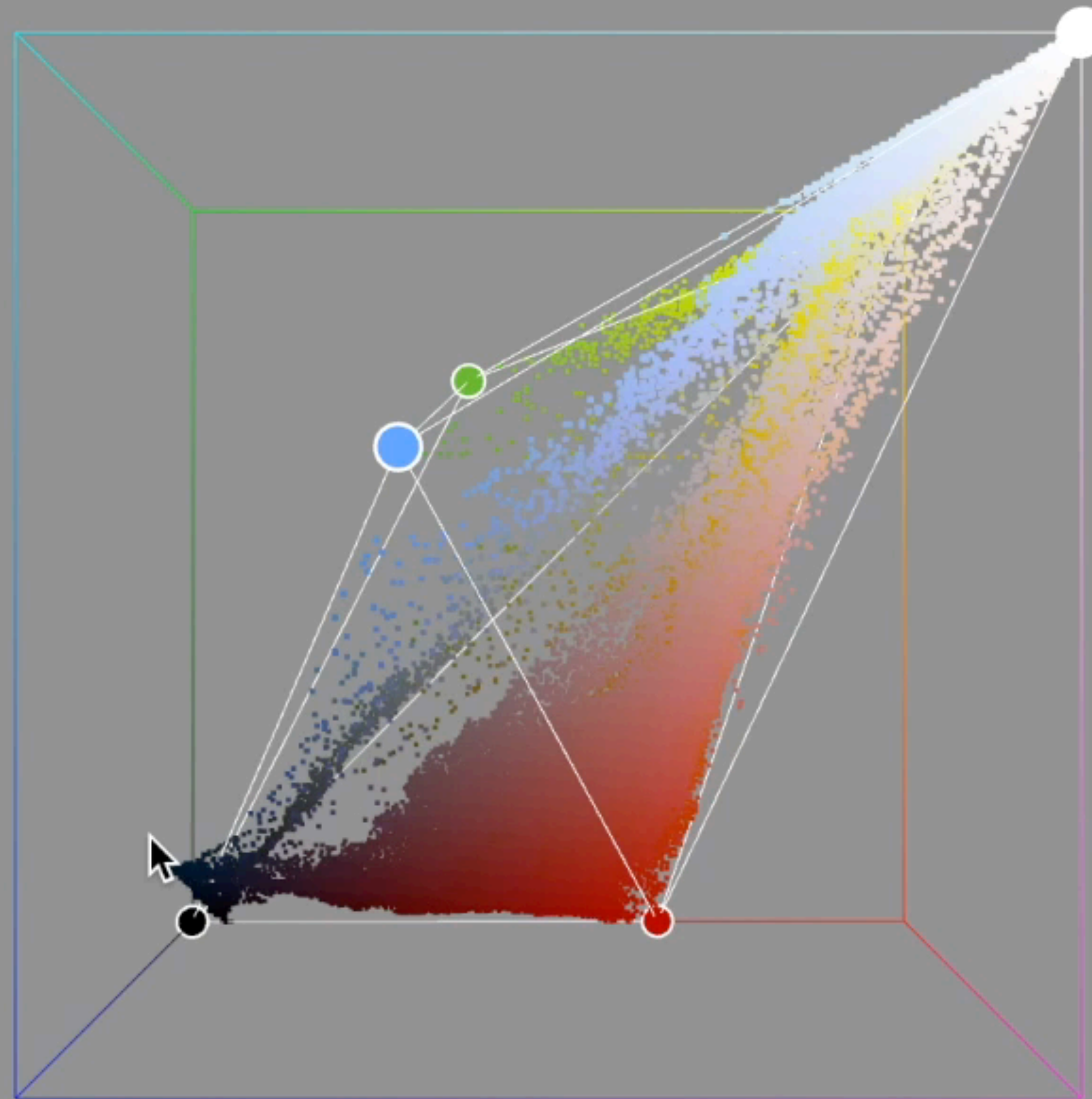
Choose File No file chosen

Rotation has inertia:

[Look from white](#)

[Save Everything](#)

[Save Camera Only](#)





Visualize the colors of an image as a 3D RGB point cloud.

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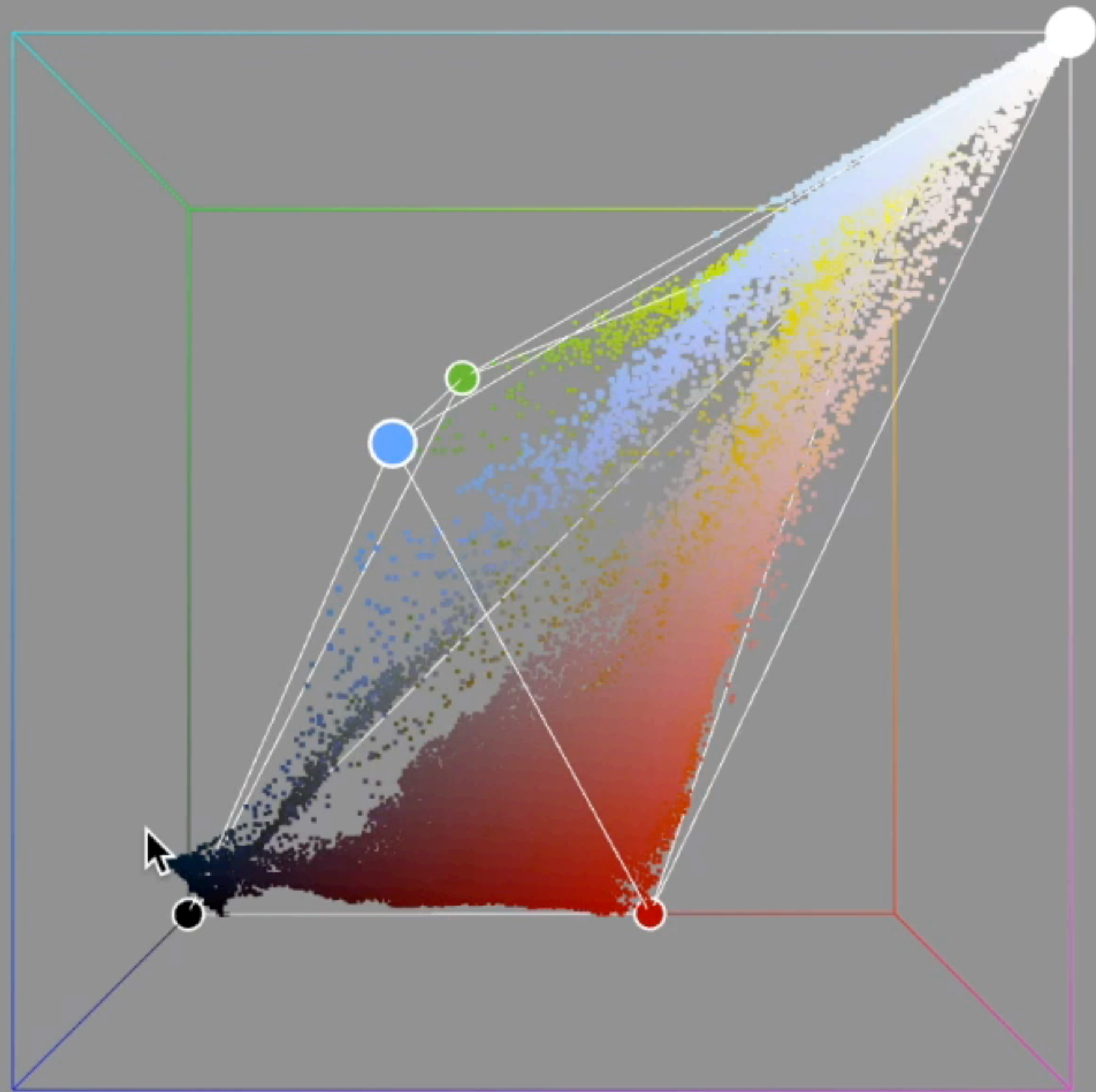
Choose File No file chosen

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[Save Camera Only](#)



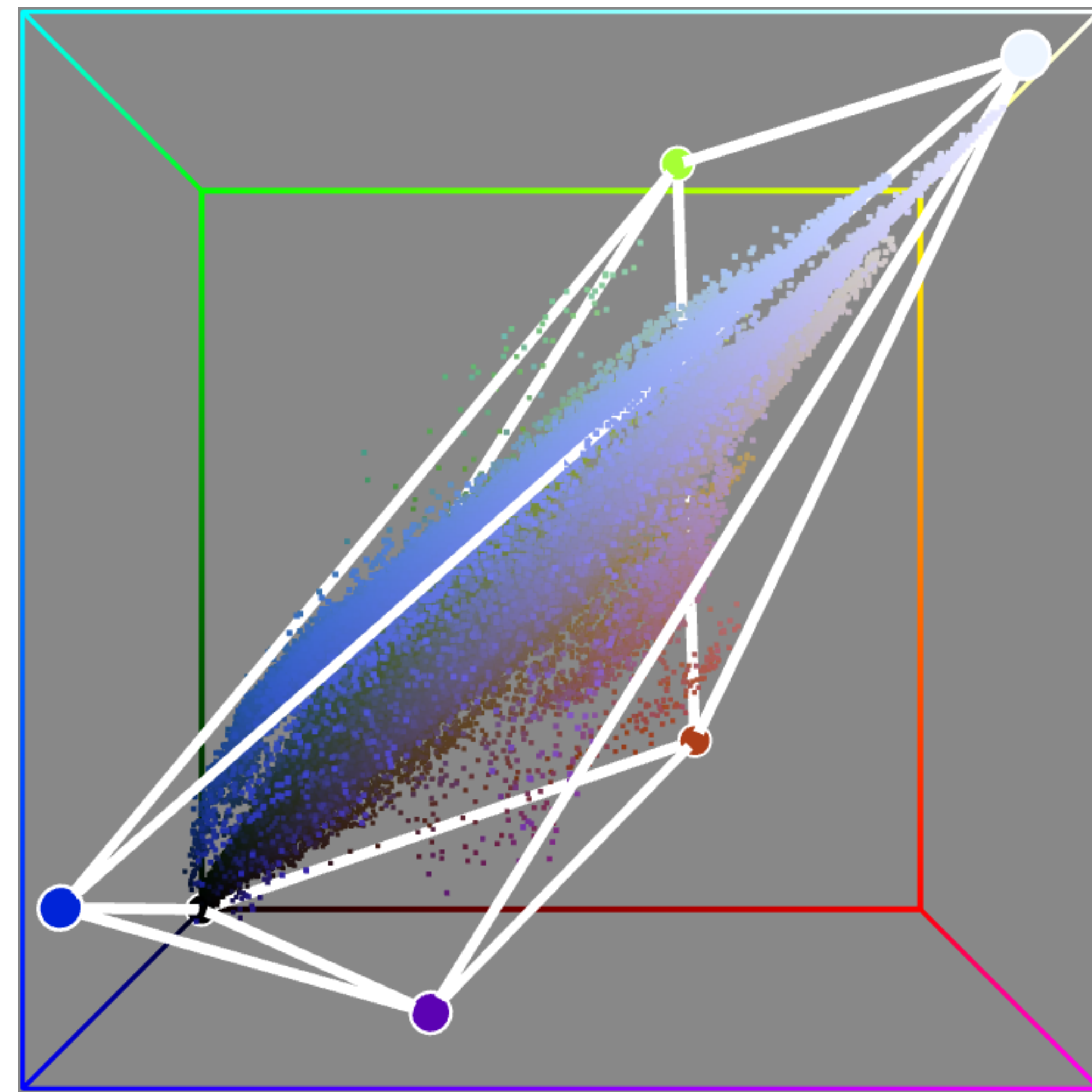


# Natural Images

*Original*



*Simplified hull*

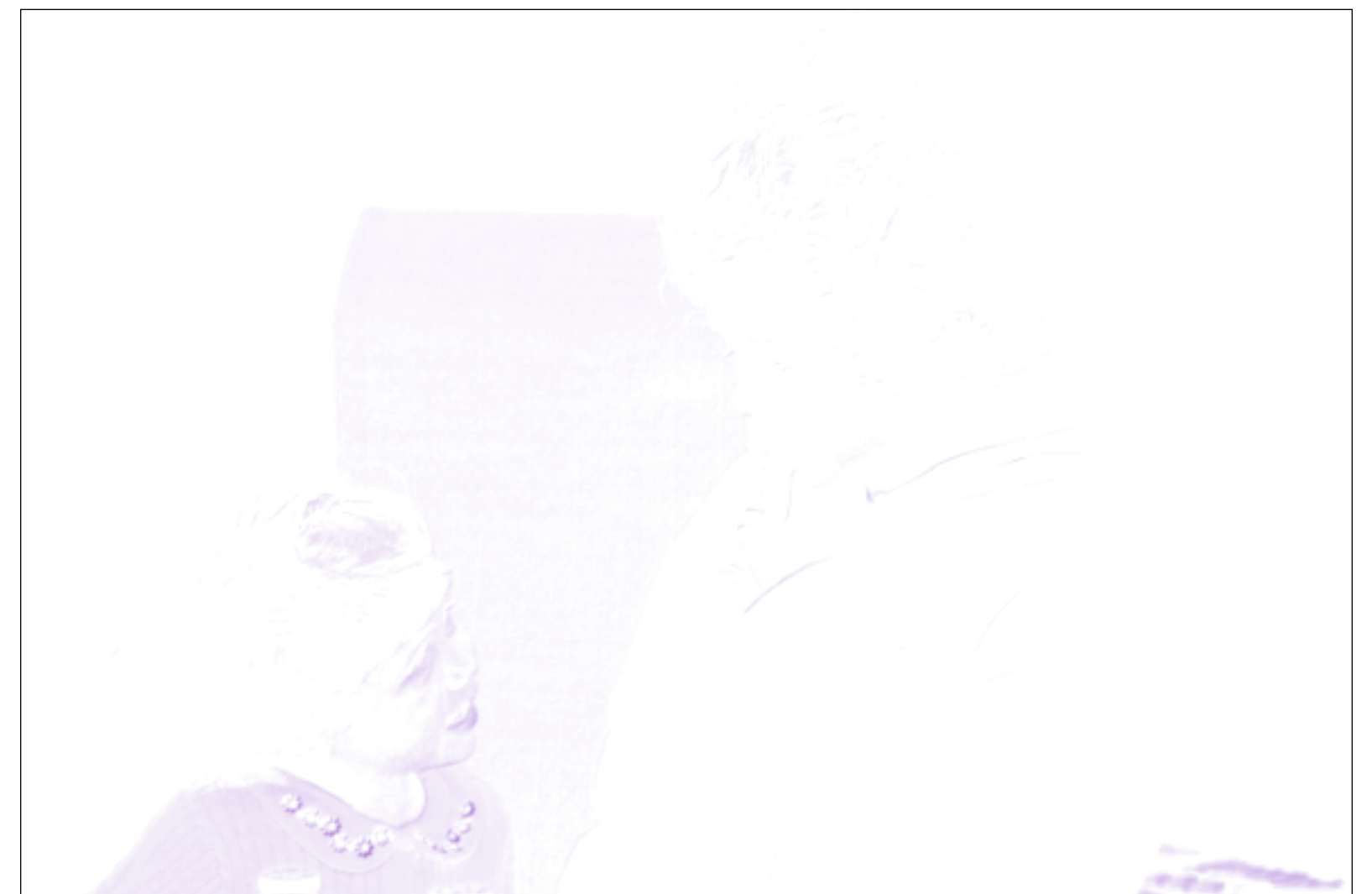
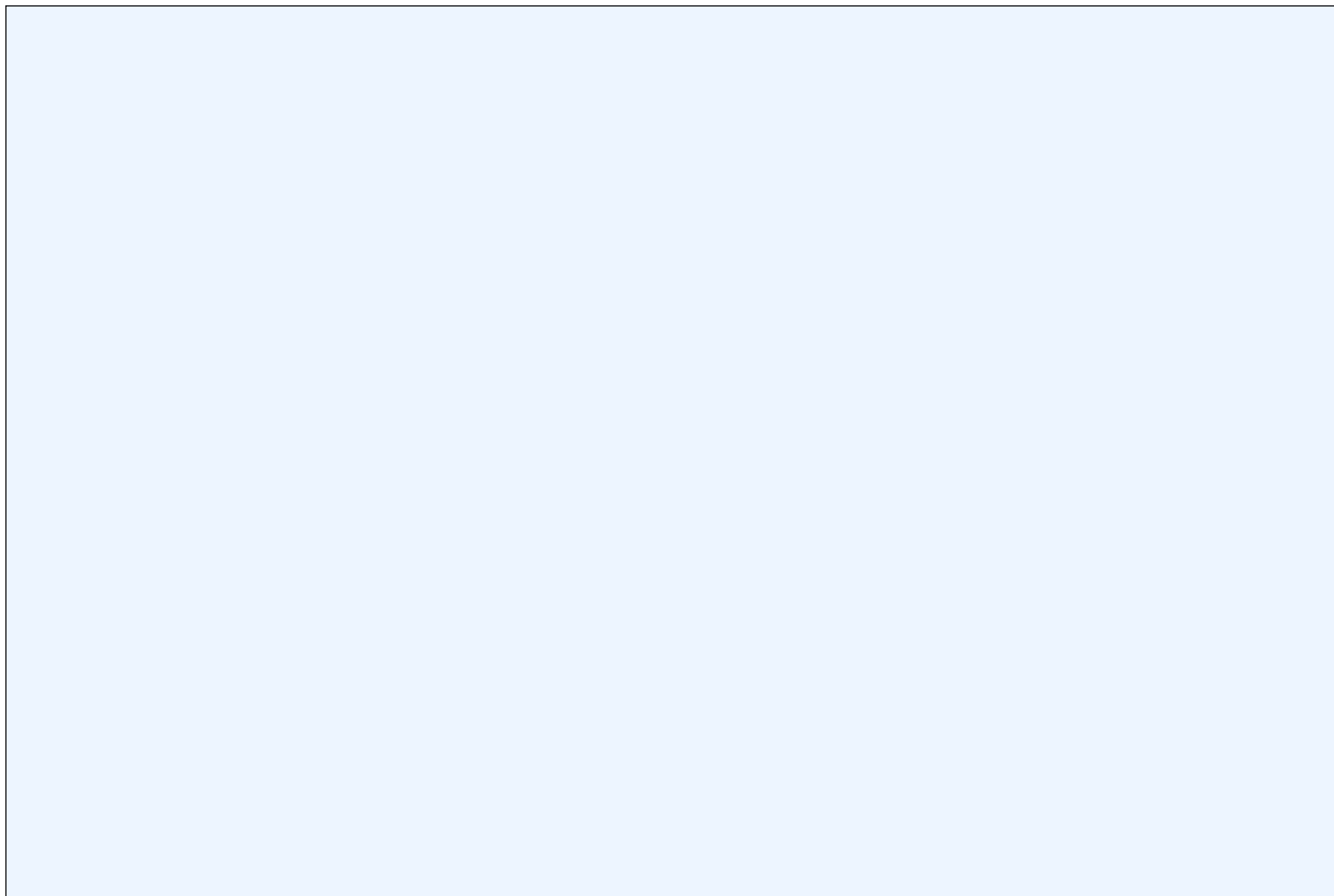








*layers*





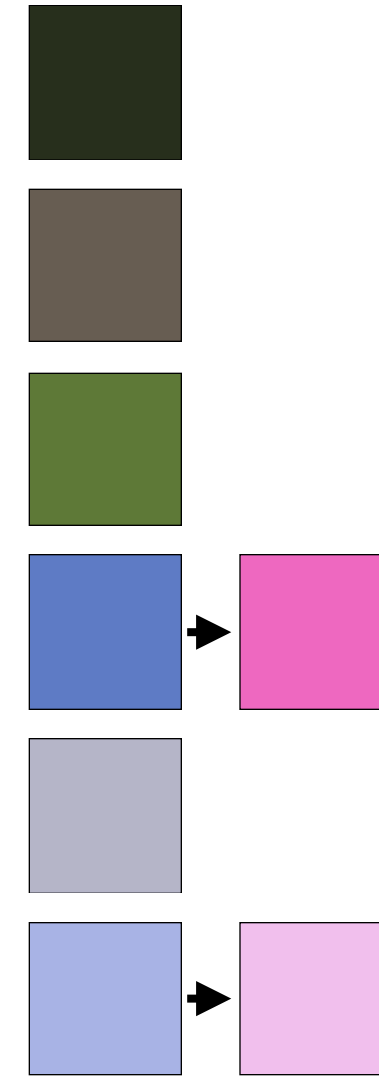




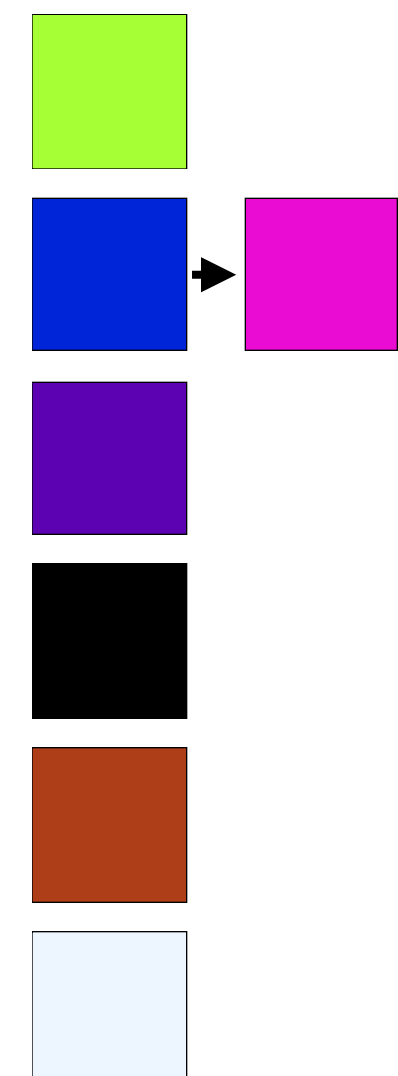
# Global Recoloring Comparison



Original



Chang et al. 2015



Ours



MVC



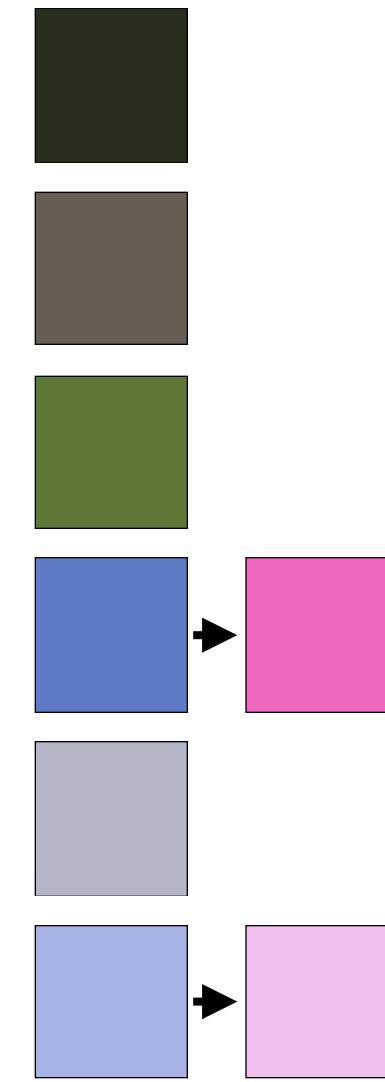
LBC



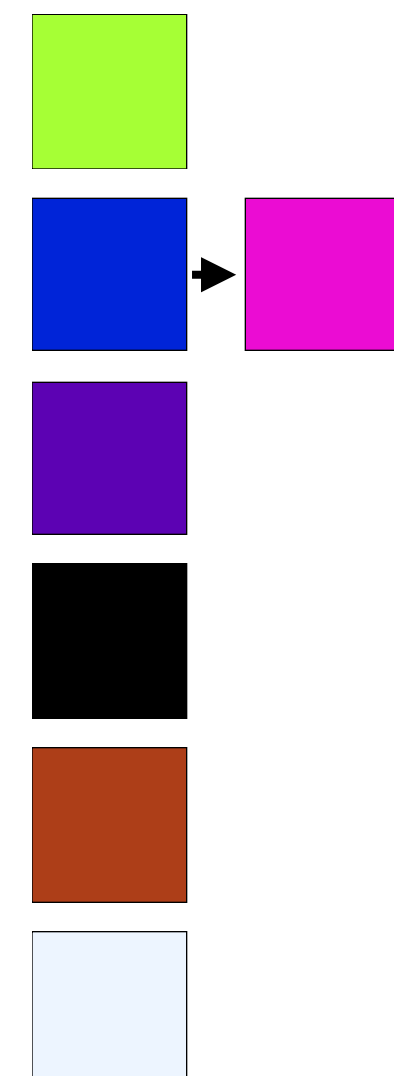
# Global Recoloring Comparison



Original



Chang et al. 2015



Ours



MVC



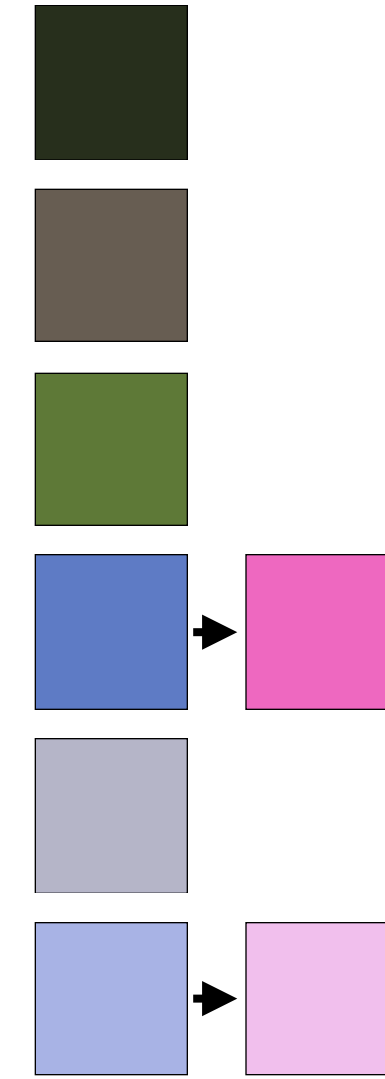
LBC



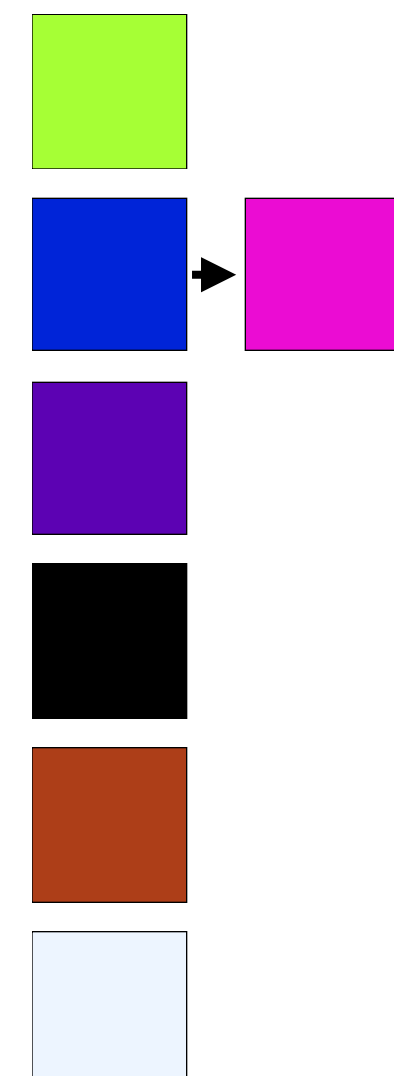
# Global Recoloring Comparison



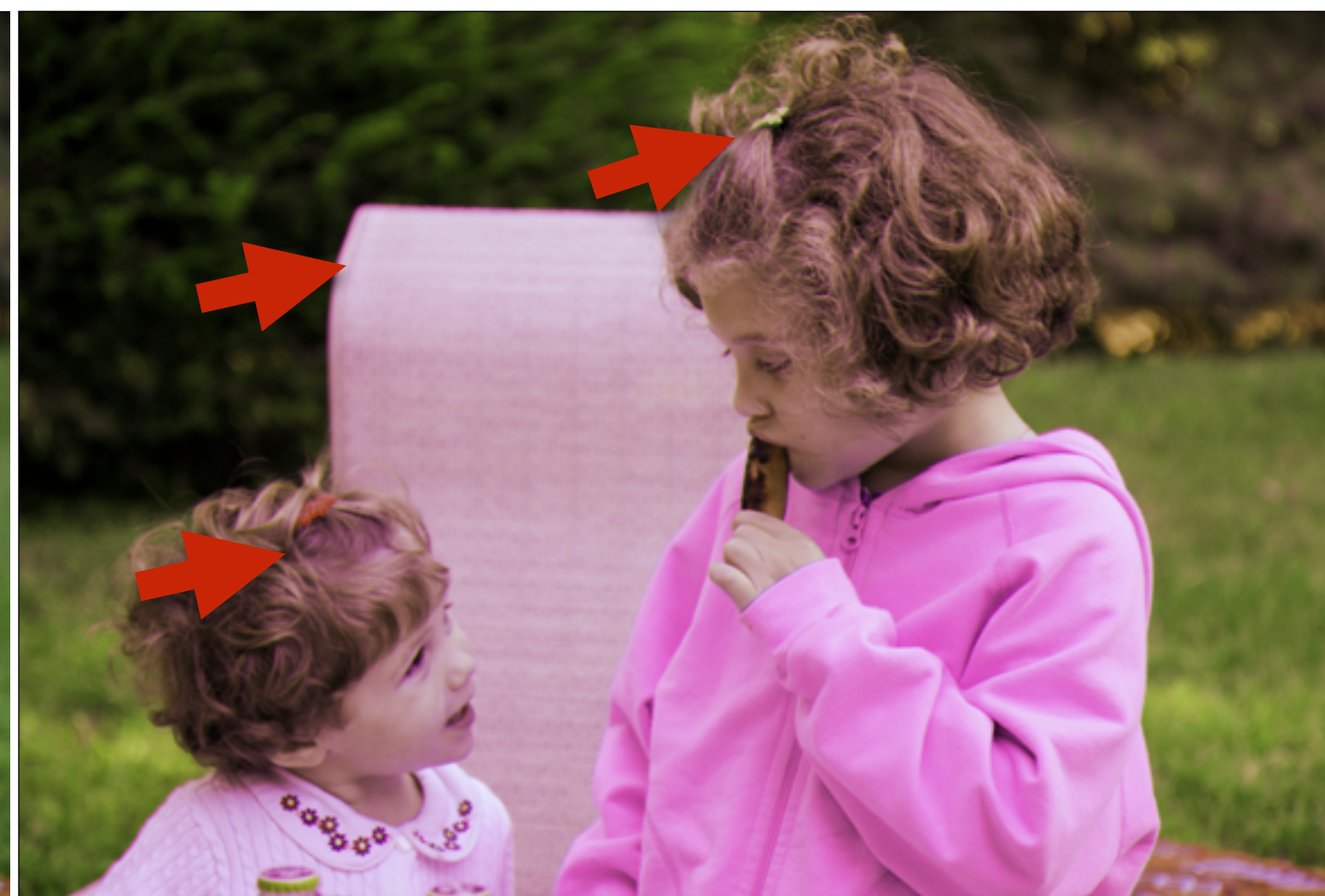
Original



Chang et al. 2015



Ours



MVC



LBC



# Global Recoloring Comparison

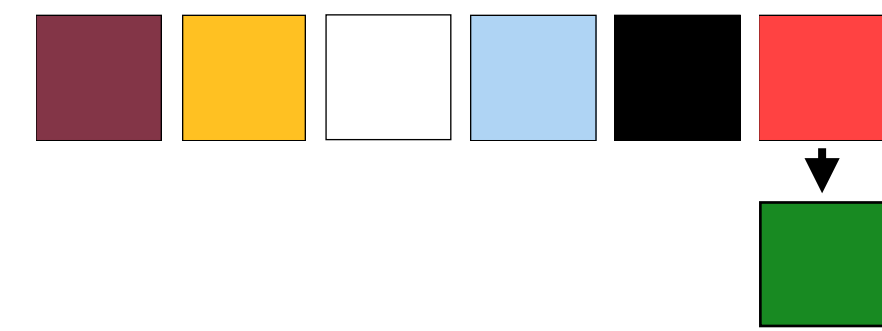
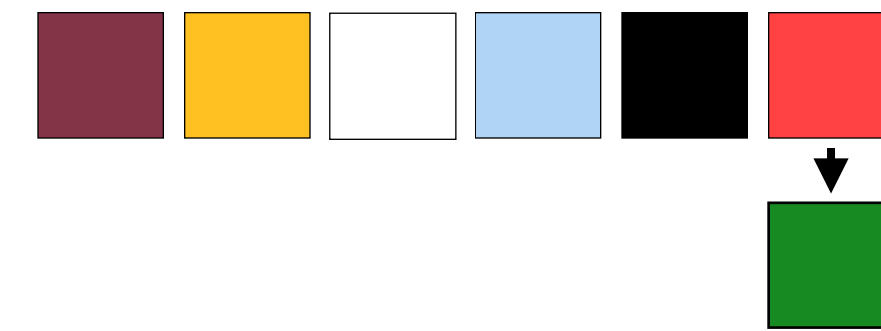
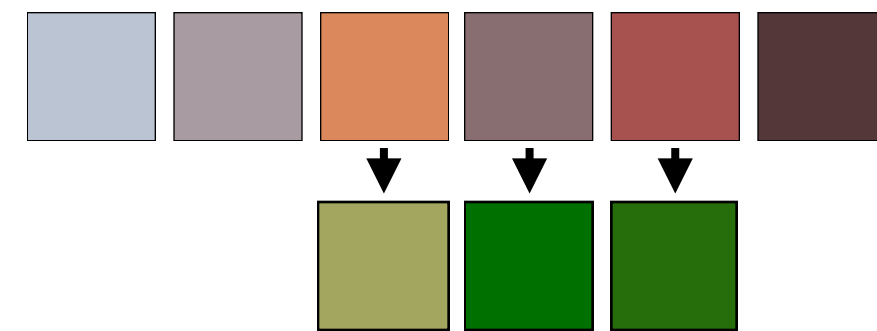
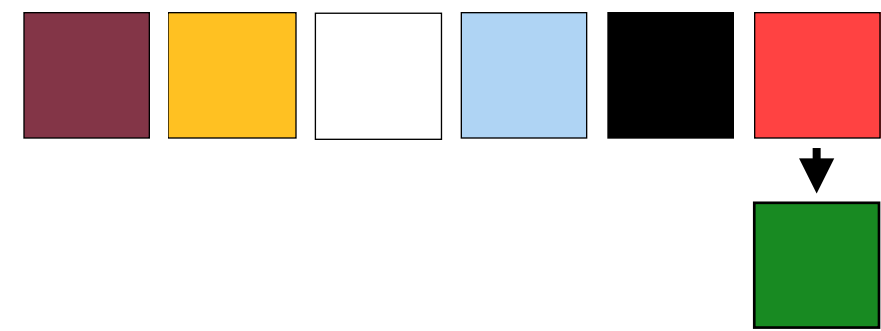
Original

Ours

Chang et al. 2015

MVC

LBC





# Global Recoloring Comparison

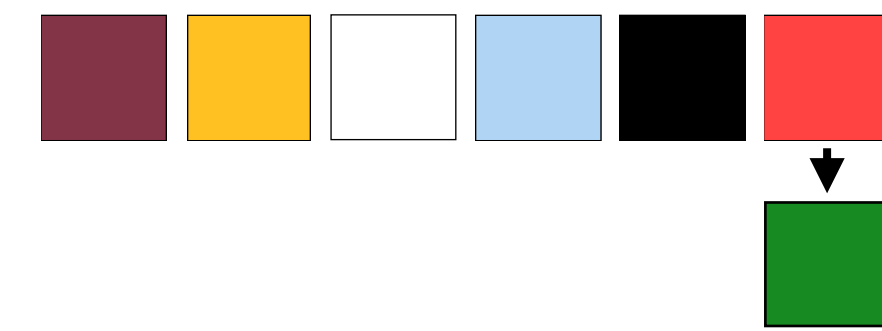
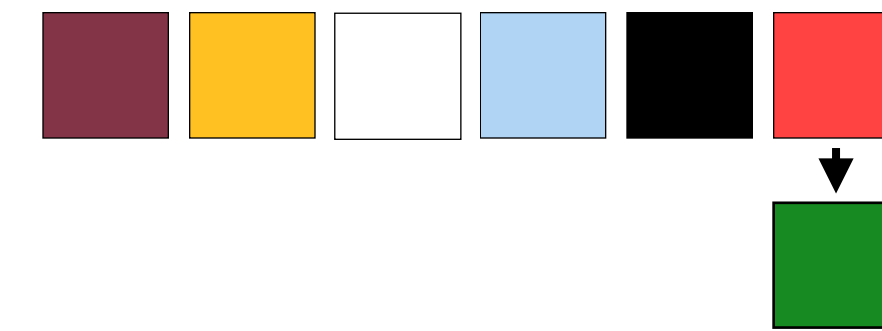
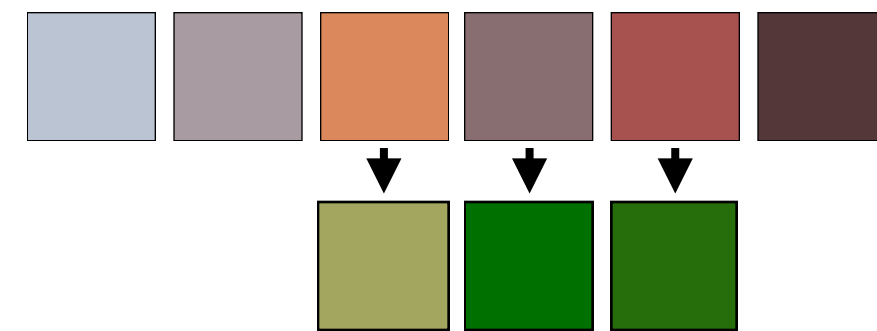
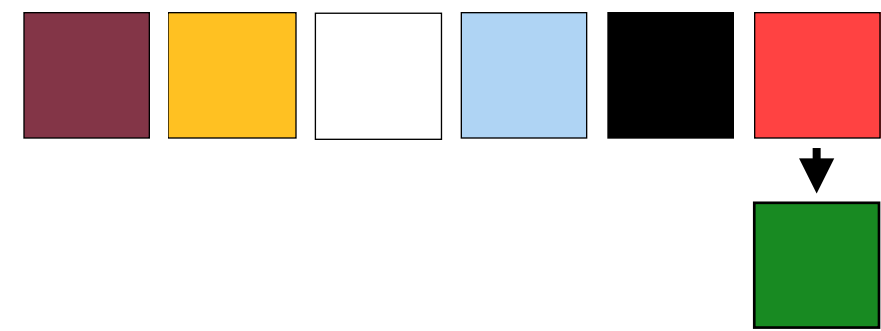
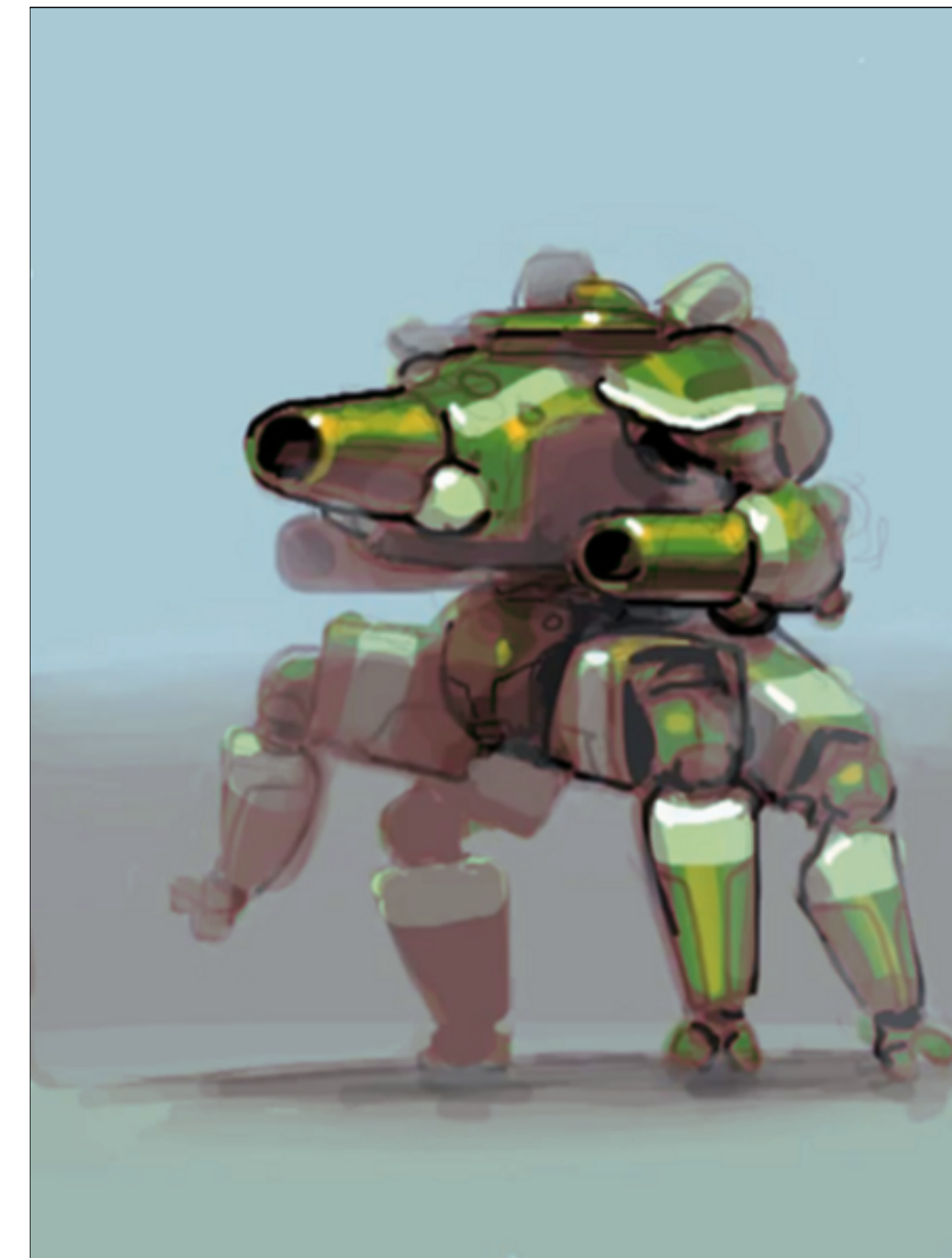
Original

Ours

Chang et al. 2015

MVC

LBC





# Global Recoloring Comparison

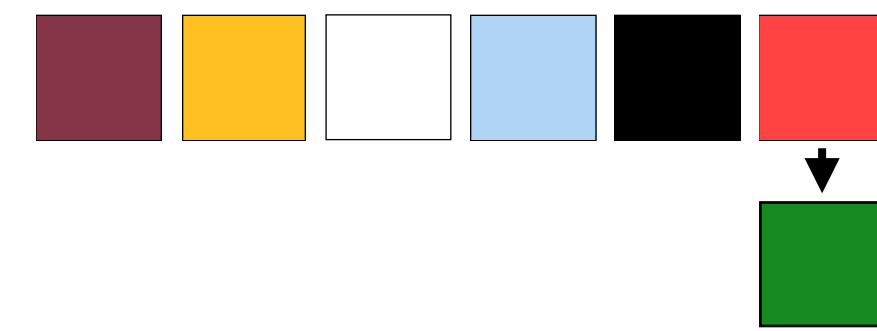
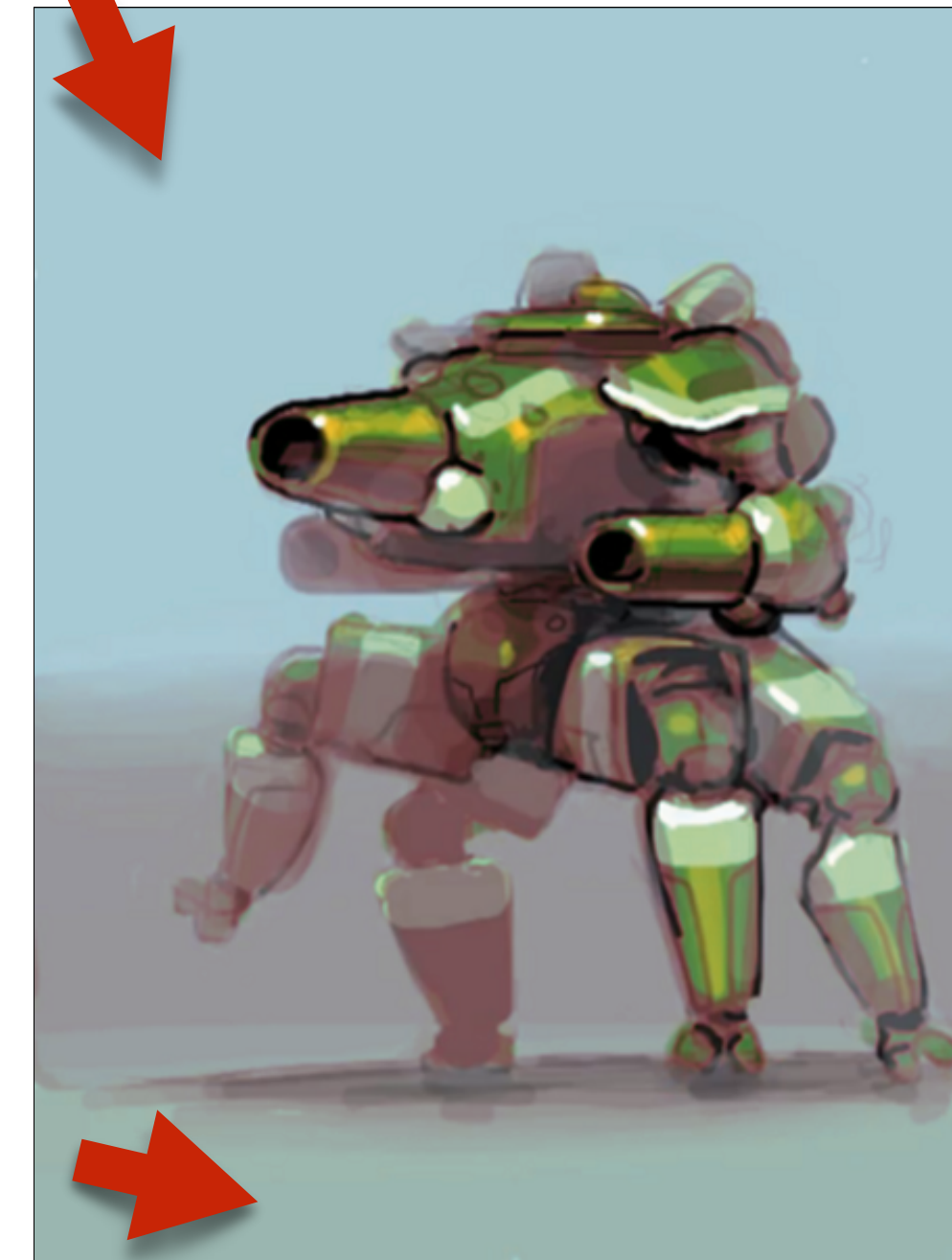
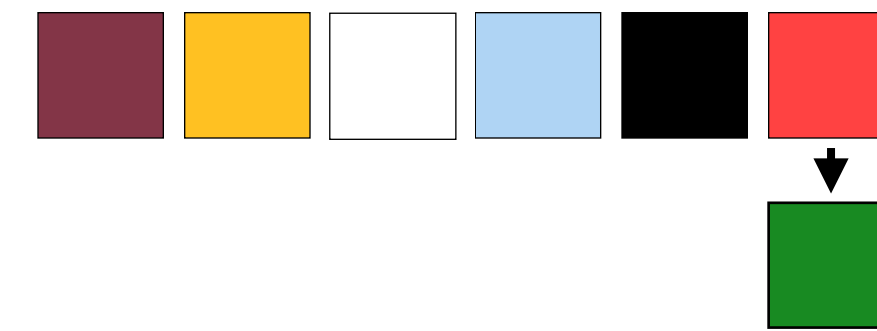
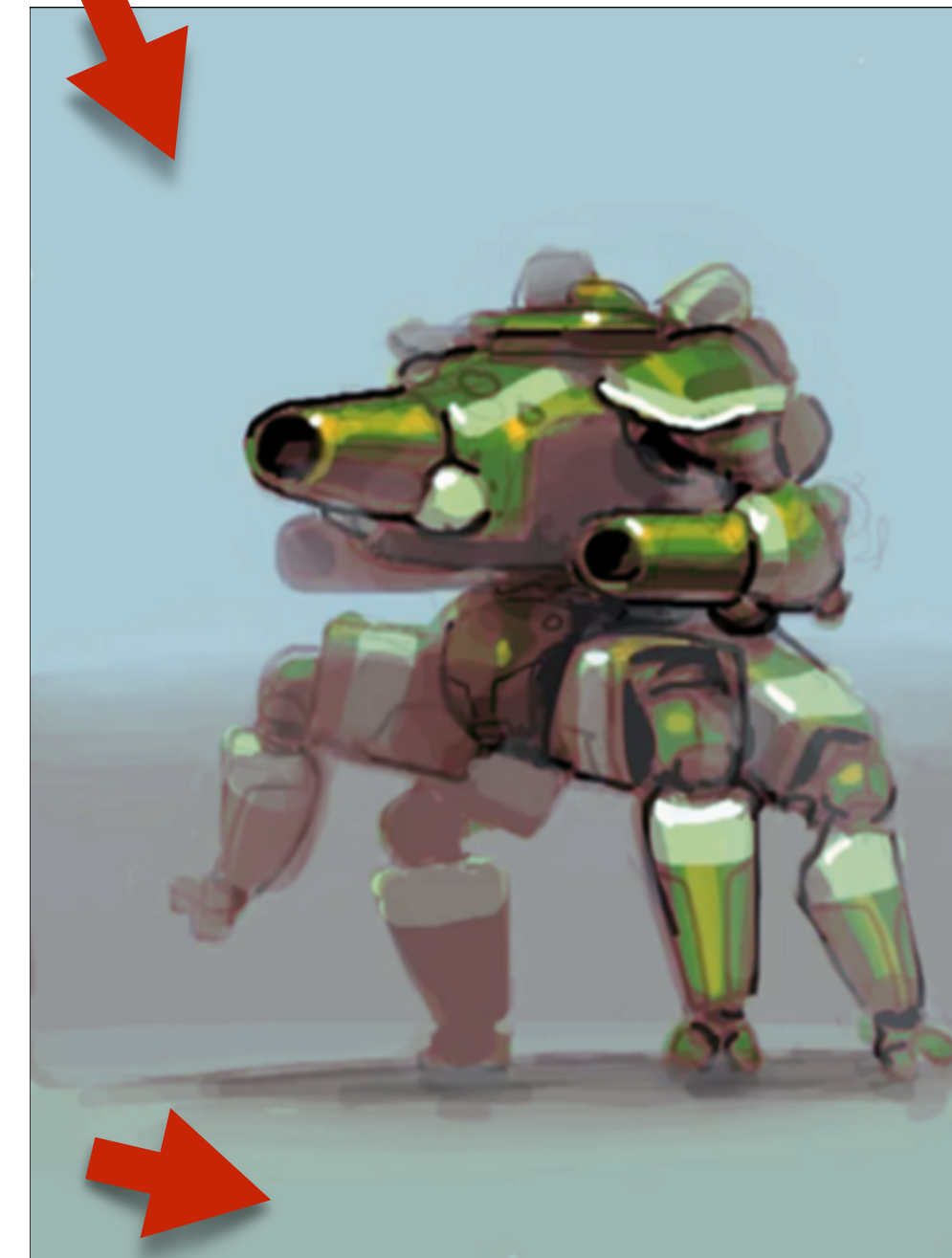
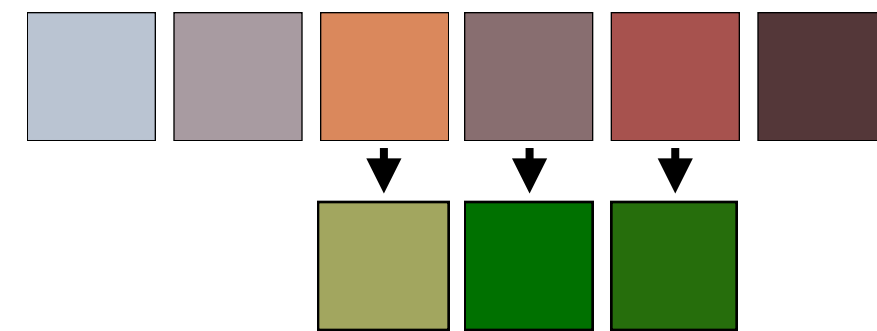
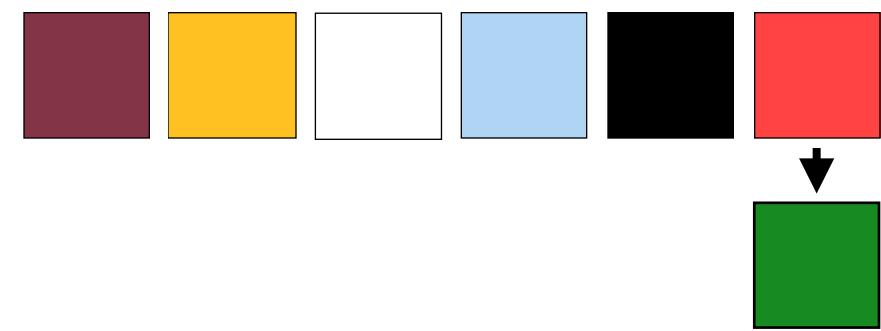
Original

Ours

Chang et al. 2015

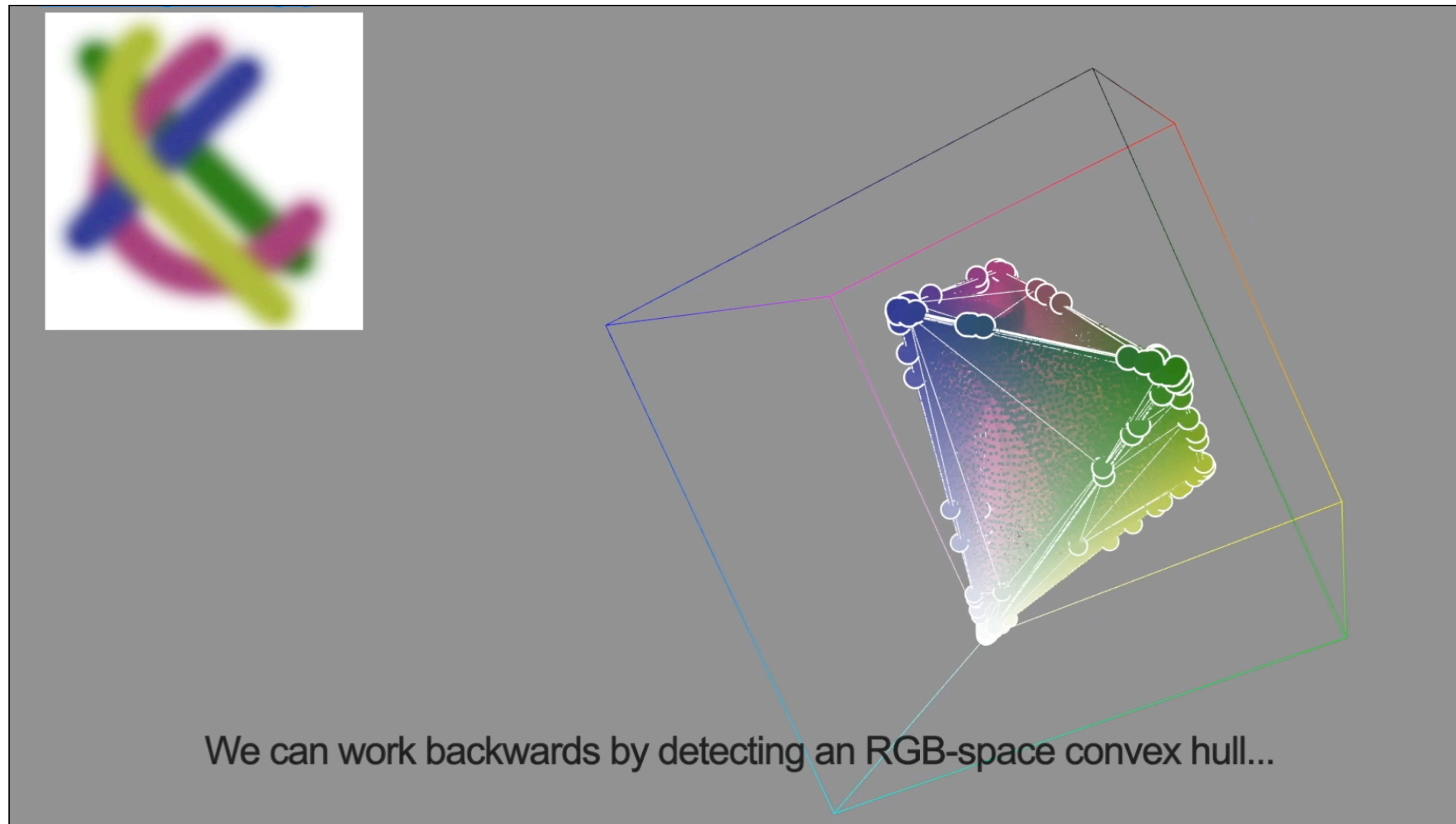
MVC

LBC



# Summary

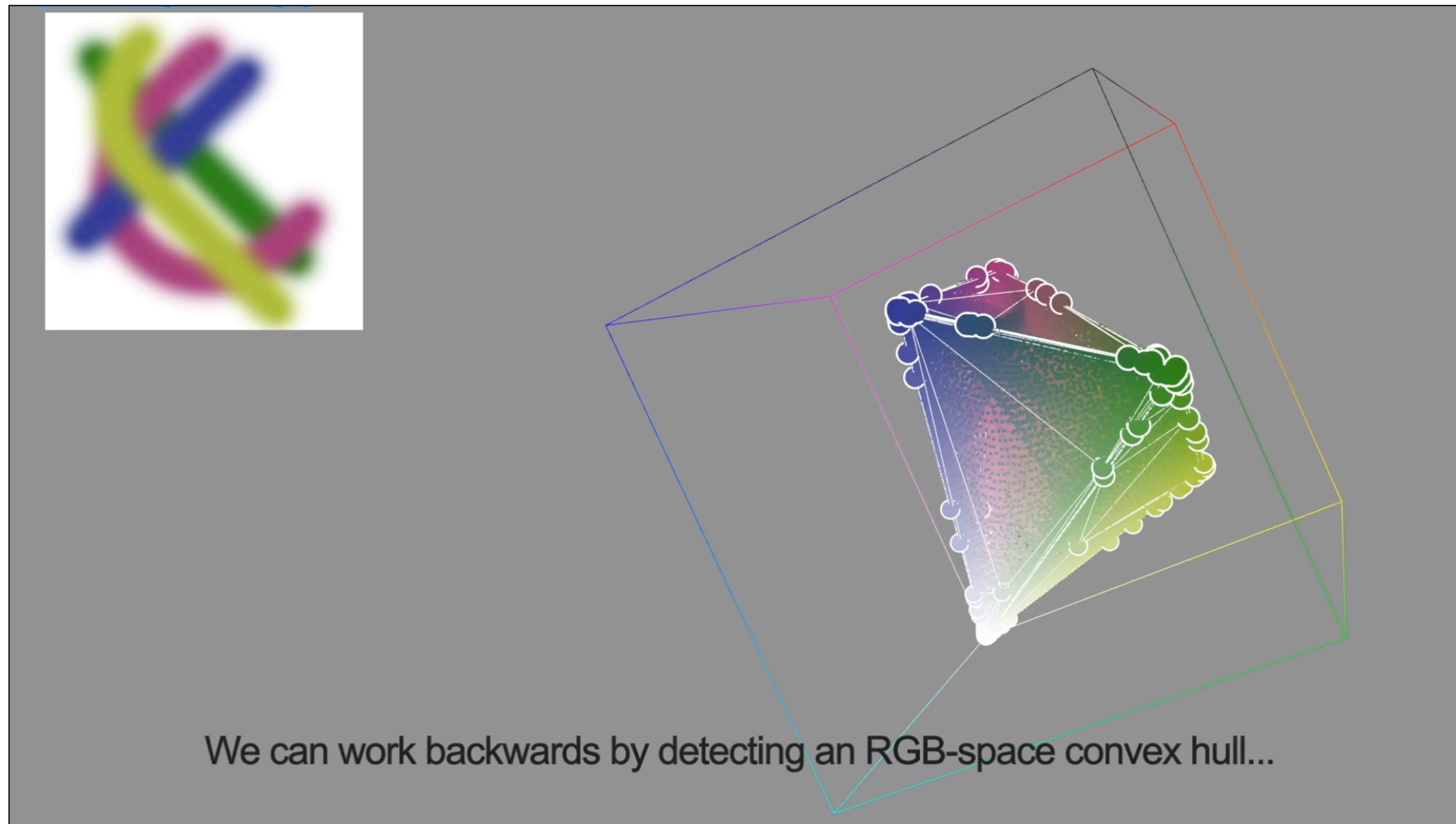
- The RGB-space geometry of an image contains a hidden geometric structure.





# Summary

- The RGB-space geometry of an image contains a hidden geometric structure.



# Summary

- We regularize the under-constrained layer opacity problem by balancing sparsity and smoothness.

**Original**

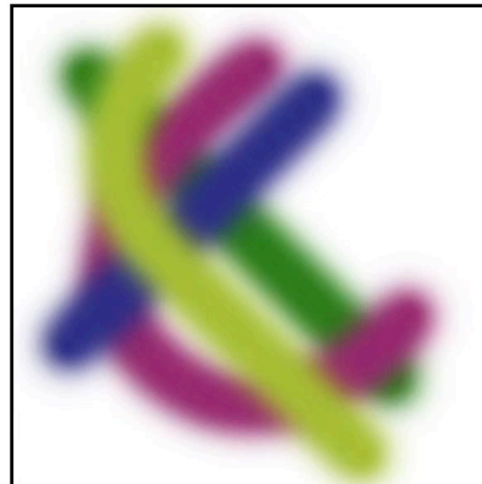


We then solve an optimization problem to extract translucent layers.

# Summary

- We regularize the under-constrained layer opacity problem by balancing sparsity and smoothness.

**Original**

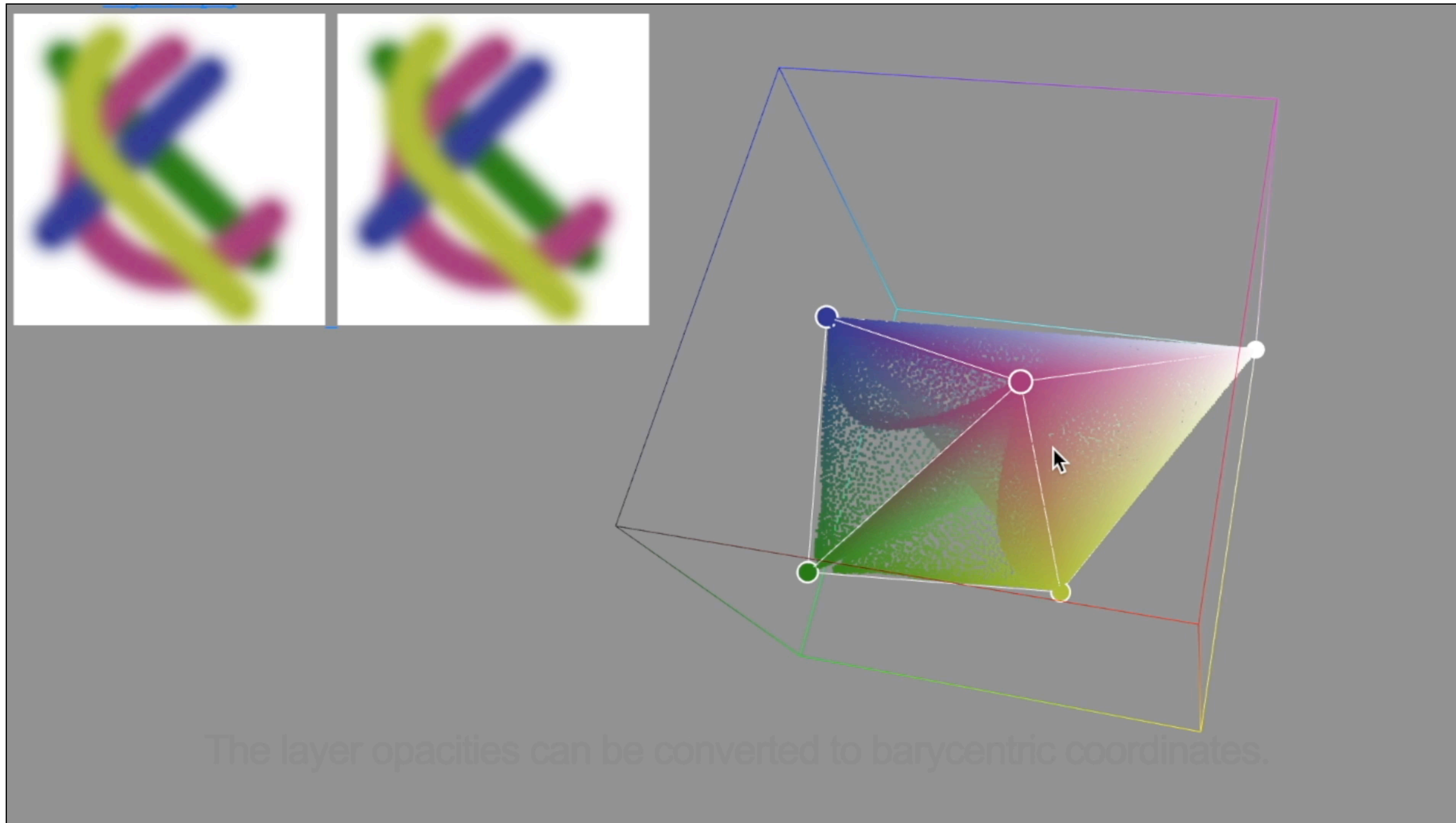


We then solve an optimization problem to extract translucent layers.



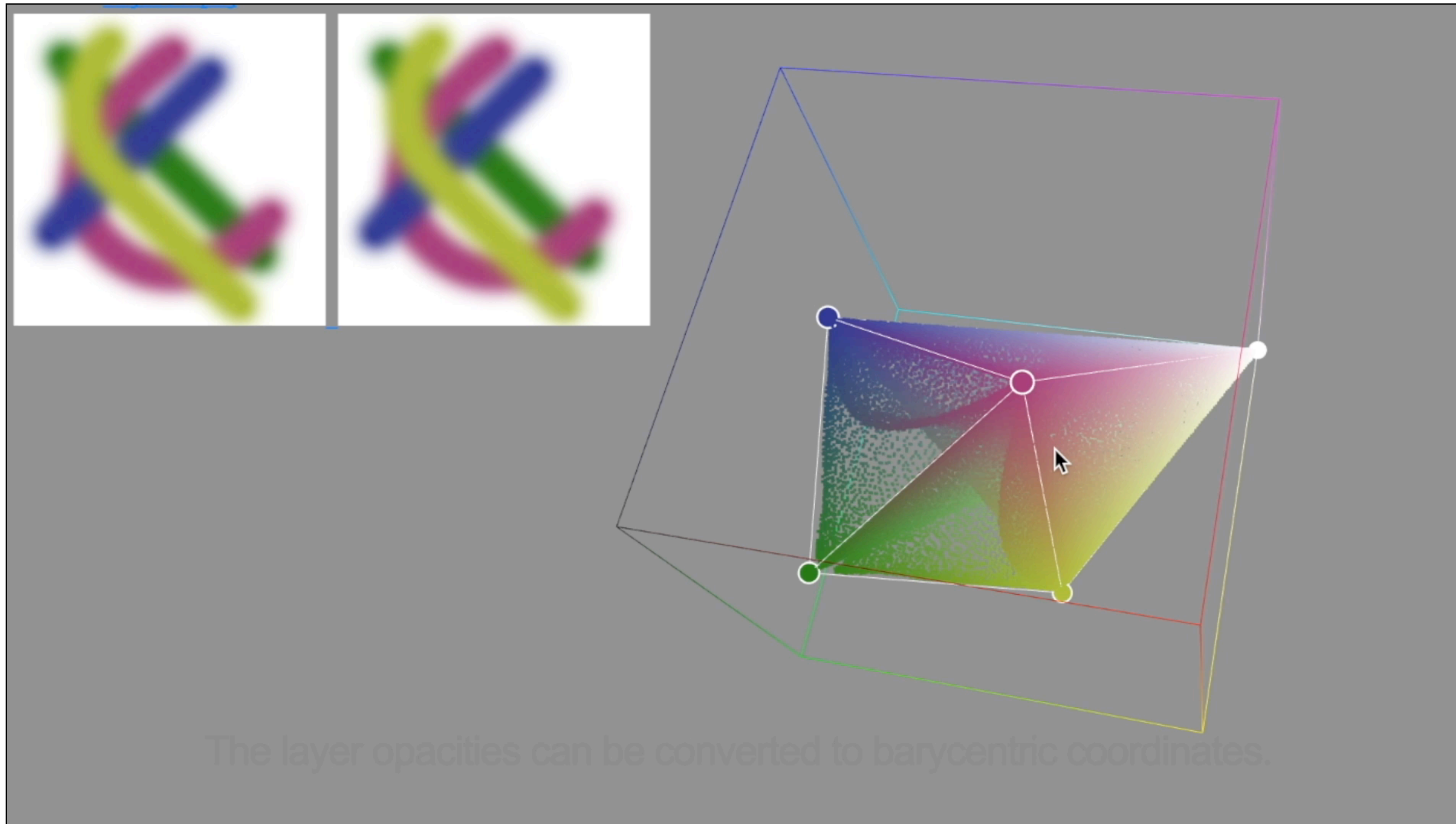
# Summary

- The layers can be edited or converted to generalized barycentric coordinates



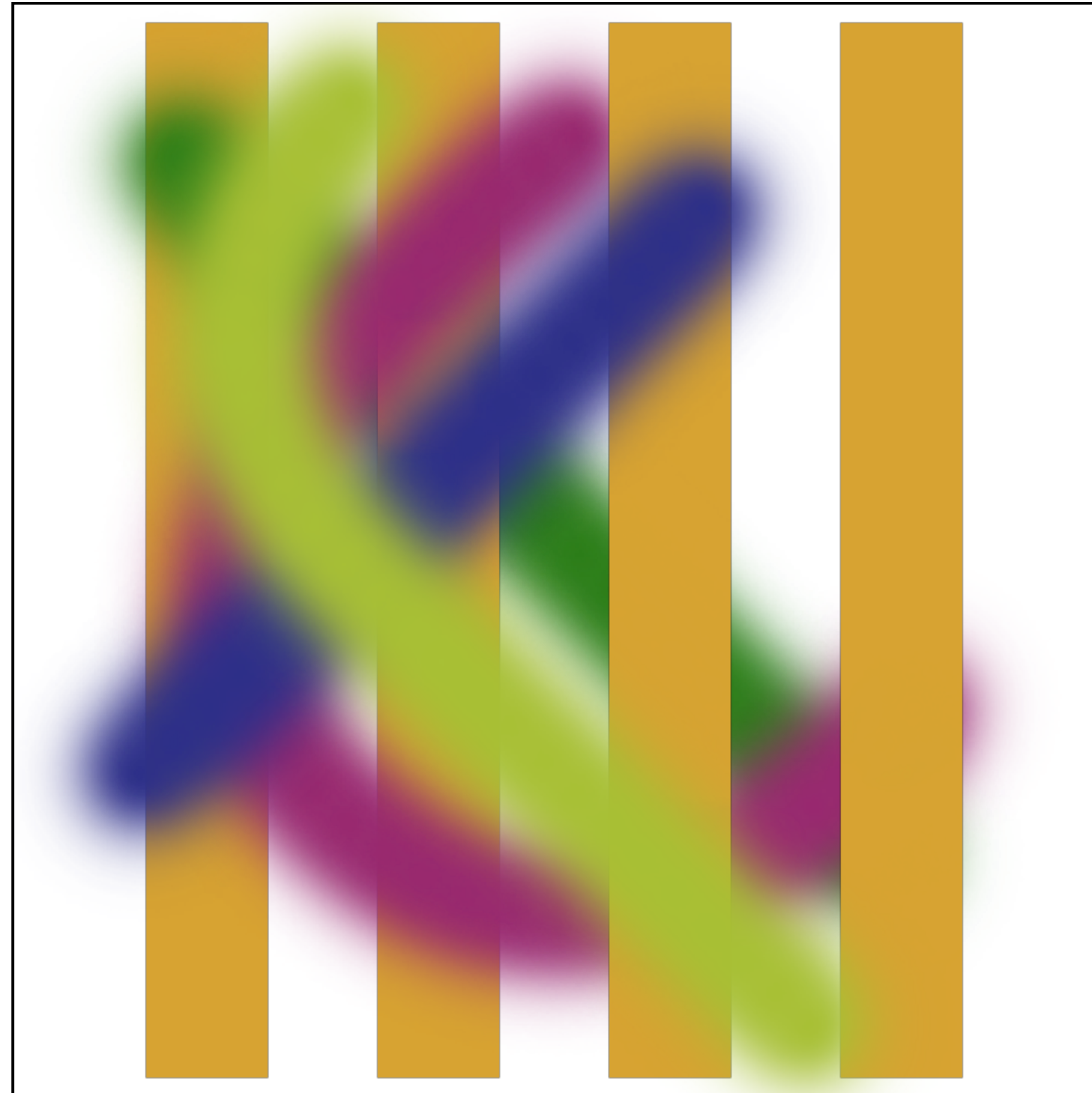
# Summary

- The layers can be edited or converted to generalized barycentric coordinates



# Limitation

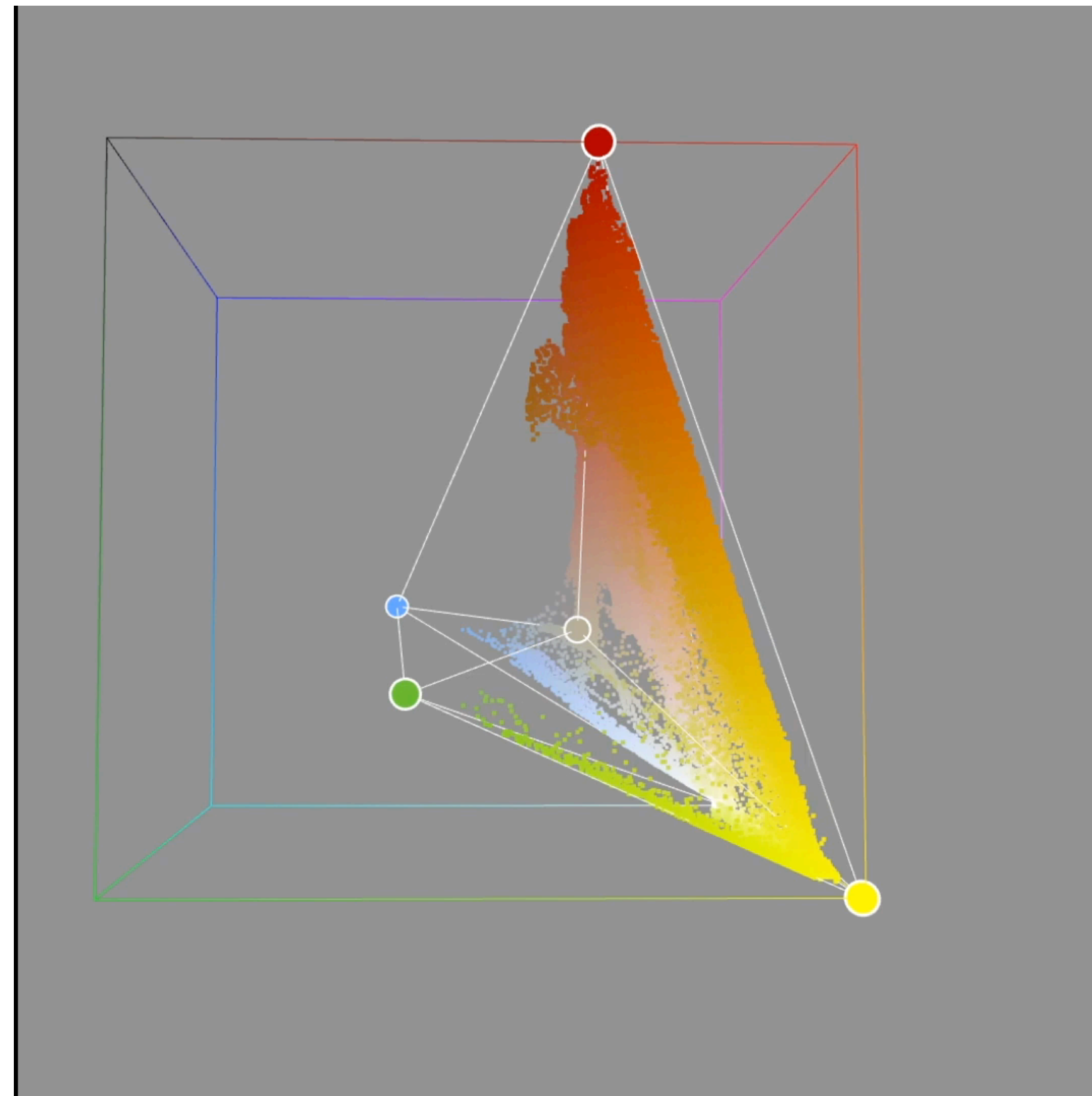
- We use a global order for layers, which may not match true editing history.





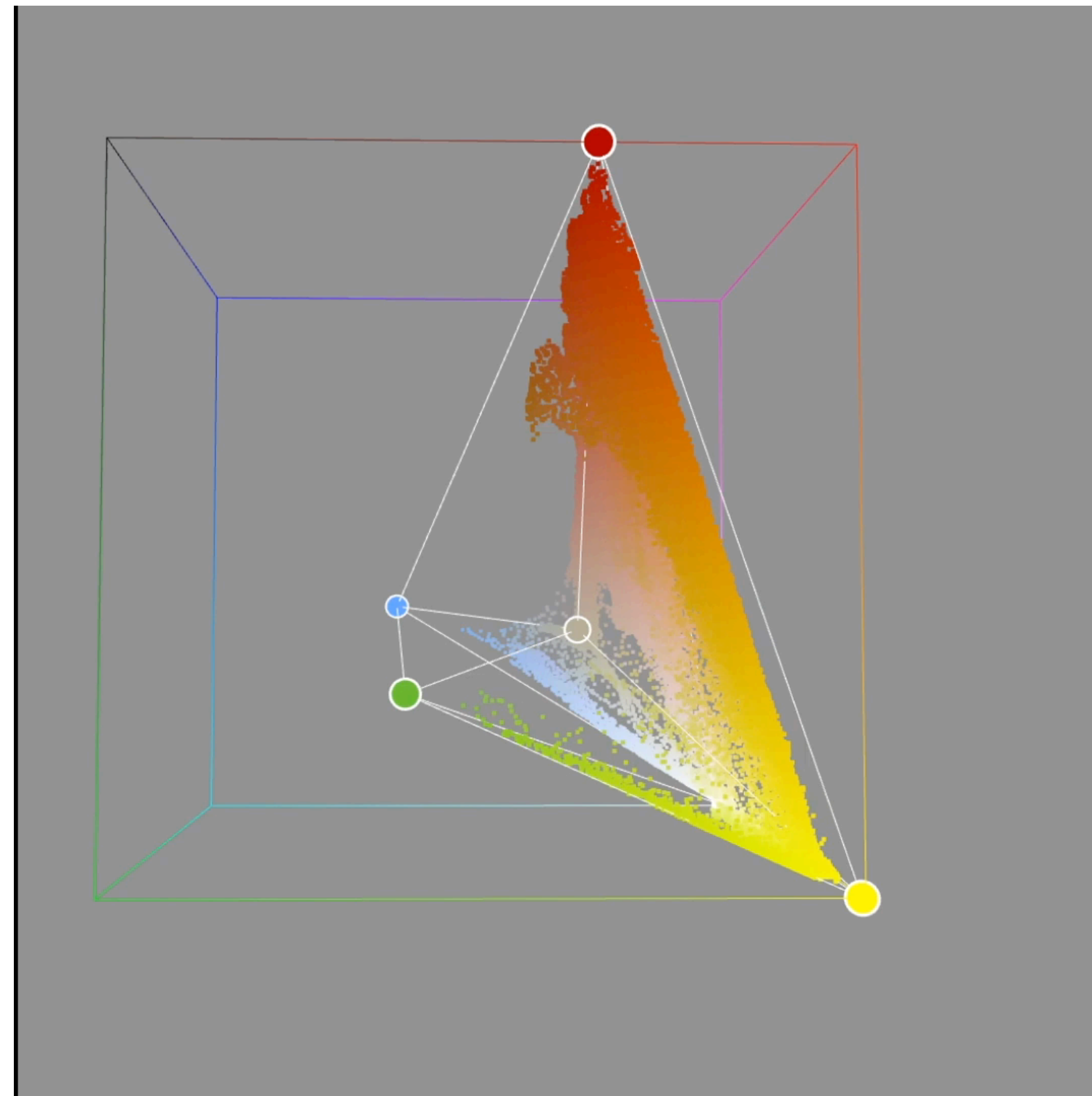
# Limitation

- Pigment colors that lie within the convex hull cannot be detected.



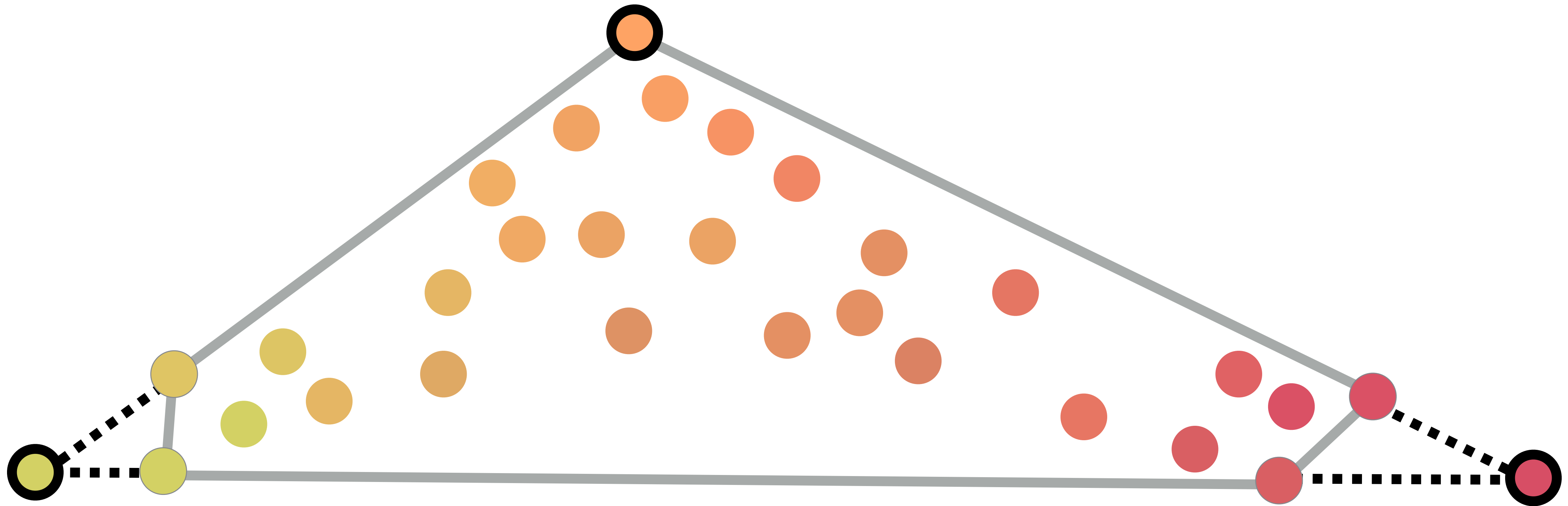
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# Limitation

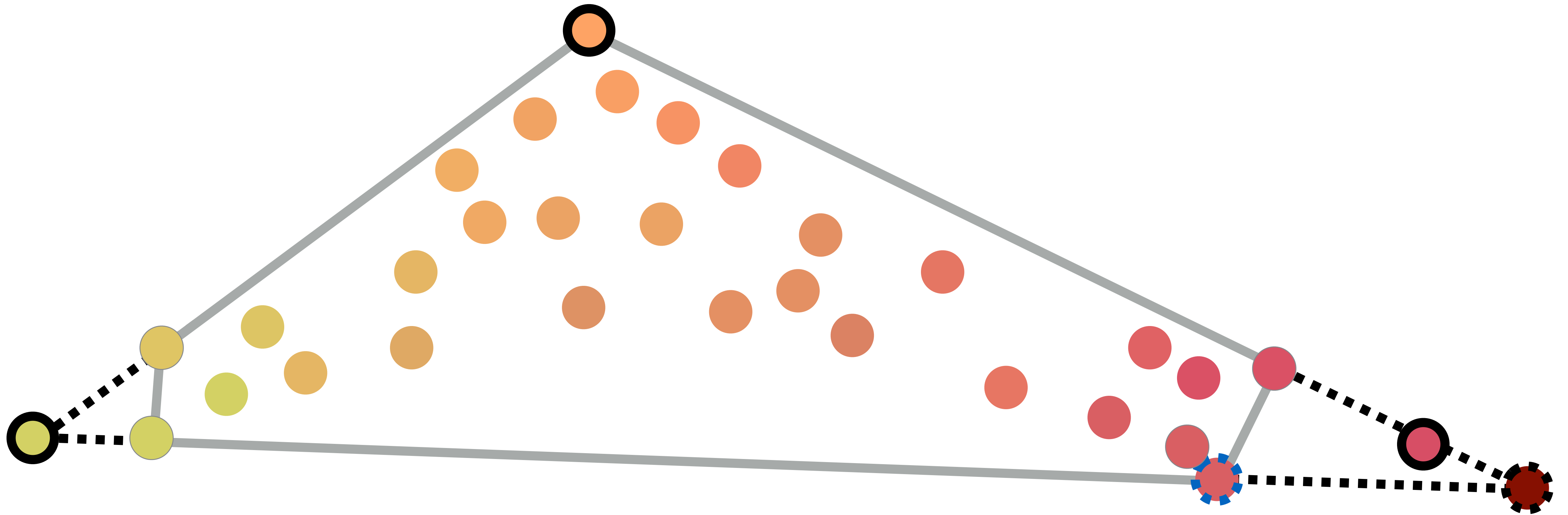
- Outlier colors can influence the convex hull used in palette selection.





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- Outlier colors can influence the convex hull used in palette selection.



# Future Work

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- More semantic palette extraction.
- Physically-inspired blending models (e.g. Kubelka-Munk).
- Additive mixing layers (works well, similar optimization but quadratic).



# Thank You!

- Contact Information
  - Jianchao Tan: [jtan8@gmu.edu](mailto:jtan8@gmu.edu)
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  - Yotam Gingold: [ygingold@gmu.edu](mailto:ygingold@gmu.edu)
- Project Website (GUI, code, data): <https://cragl.cs.gmu.edu/singleimage/>
- Artists: Adelle Chudleigh; Dani Jones; Karl Northfell; Michelle Lee; Adam Saltsman; Yotam Gingold.
- Sponsors:
  - United States National Science Foundation, Google.

# **Extra Slides**

## opacity optimization

image	width $\times$ height	runtime (seconds)	RMSE	median error	max error	
apple	500 $\times$ 453	330.3		1.8	0.0	32.3
bird	640 $\times$ 360	500.4		3.7	2.2	60.1
rowboat	589 $\times$ 393	981.4		3.3	2.4	19.2
buildings	589 $\times$ 393	317.9		2.3	1.4	32.2
cup	400 $\times$ 400	40.9		3.0	0.0	34.3
fruit	650 $\times$ 414	212.1		1.9	0.0	22.6
girls	589 $\times$ 393	152.7		2.4	1.4	29.1
hoover	500 $\times$ 500	47.1		3.9	1.0	39.2
light	504 $\times$ 538	101.6		2.4	1.0	25.5
robot	450 $\times$ 600	519.8		3.5	2.8	14.5
scrooge	410 $\times$ 542	97.6		2.6	1.4	15.7
Figure 4	500 $\times$ 500	434.3		0.7	0.0	3.3
trees	606 $\times$ 404	80.2		5.9	4.2	35.0
turtle	525 $\times$ 250	19.5		2.8	1.0	45.4
boat	480 $\times$ 600	520.0		4.2	2.2	40.2
castle	747 $\times$ 344	280.0		3.7	2.2	45.6
turquoise	480 $\times$ 585	498.7		2.0	1.4	17.8
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## Increase solver's tolerance values

## Runtime

## RMSE

39	1.9
68	3.8
140	4.2
54	2.3
21	2.8
38	2.0
49	2.7
29	3.9
46	2.3
50	3.7
38	2.6
37	1.5
53	5.8
15	2.8
105	2.2
66	3.6
129	2.7
67	3.3

## opacity optimization

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Much  
faster!

# Decompose Image into layers

Solve an optimization problem, with some regularization terms



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Solve an optimization problem, with some regularization terms

$$E = w_{polynomial} E_{polynomial} + w_{opaque} E_{opaque} + w_{spatial} E_{spatial}$$

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$$E_{polynomial} = \left\| \mathbf{c}_n - \mathbf{p} + \sum_{i=1}^n \left[ (\mathbf{c}_{i-1} - \mathbf{c}_i) \prod_{j=i}^n (1 - \alpha_j) \right] \right\|^2$$

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Solve an optimization problem, with some regularization terms

$$E = w_{polynomial} E_{polynomial} + w_{opaque} E_{opaque} + w_{spatial} E_{spatial}$$

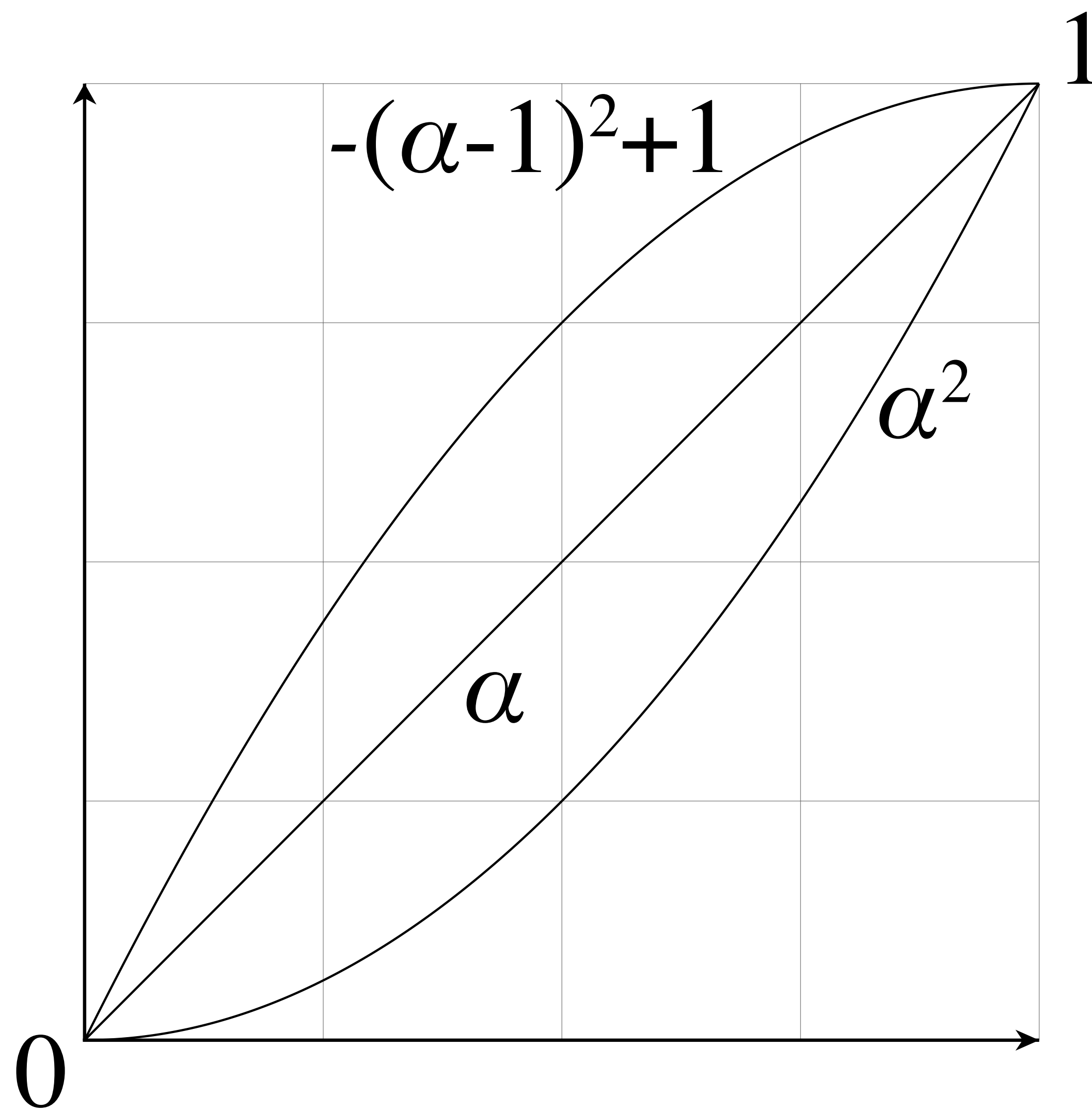
$$E_{polynomial} = \left\| \mathbf{c}_n - \mathbf{p} + \sum_{i=1}^n \left[ (\mathbf{c}_{i-1} - \mathbf{c}_i) \prod_{j=i}^n (1 - \alpha_j) \right] \right\|^2$$

$$E_{opaque} = \frac{1}{n} \sum_{i=1}^n (1 - \alpha_i)^2$$

$$E_{spatial} = \frac{1}{n} \sum_{i=1}^n (\nabla \alpha_i)^2$$

$$w_{polynomial} = 375, w_{opaque} = 1, w_{spatial} = 100.$$

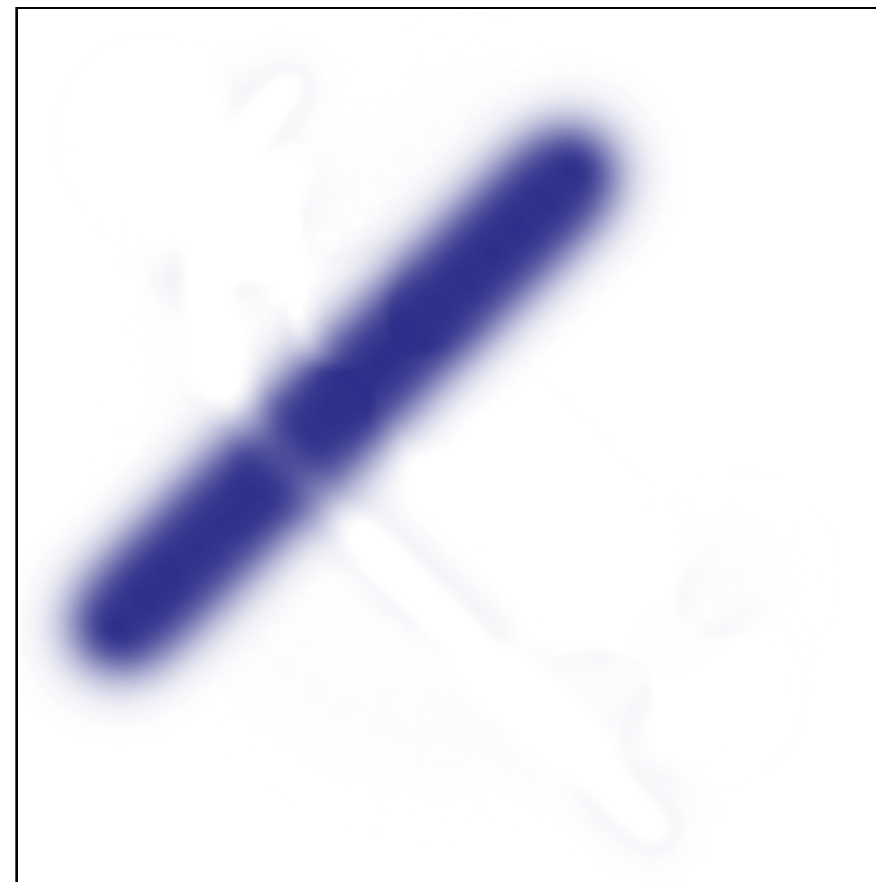
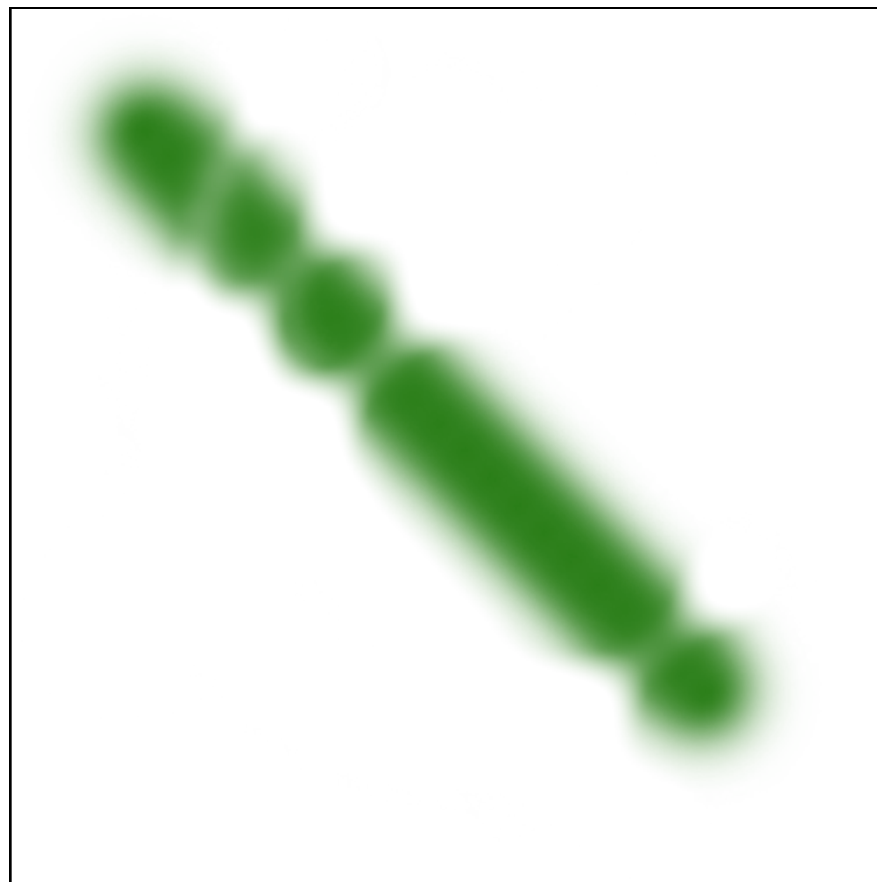
# Our sparse regularization term



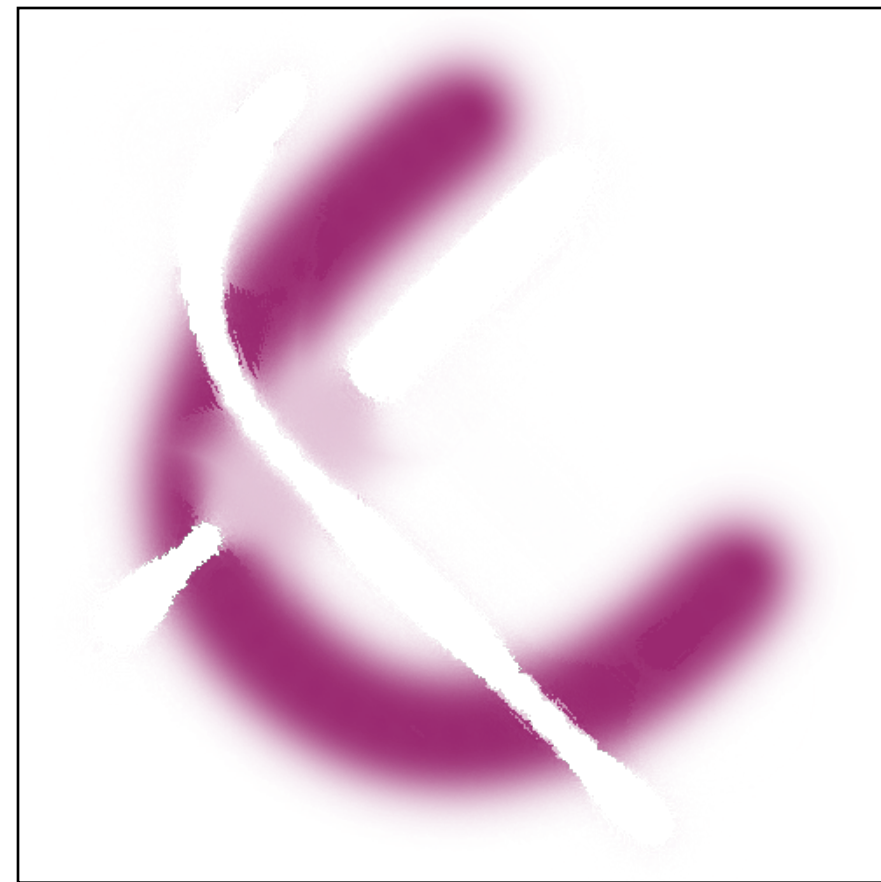
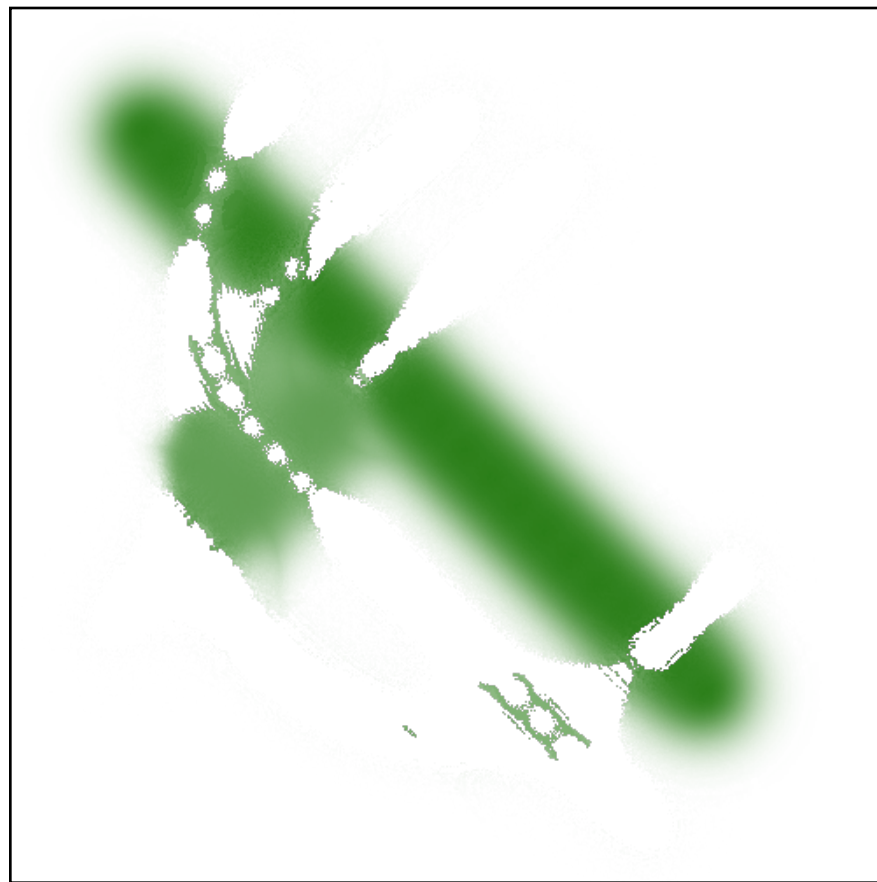


# As-sparse-as-possible(ASAP)

Ours

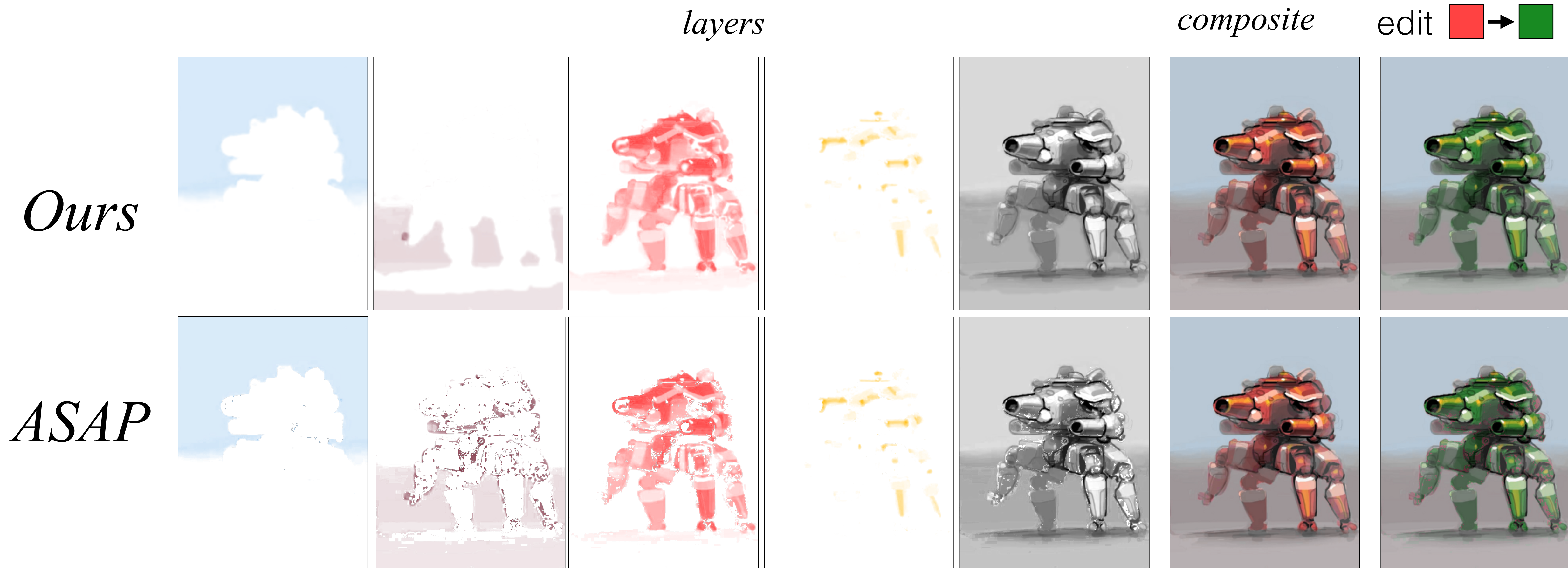


ASAP





# As-sparse-as-possible(ASAP)



# optimization parameters influence

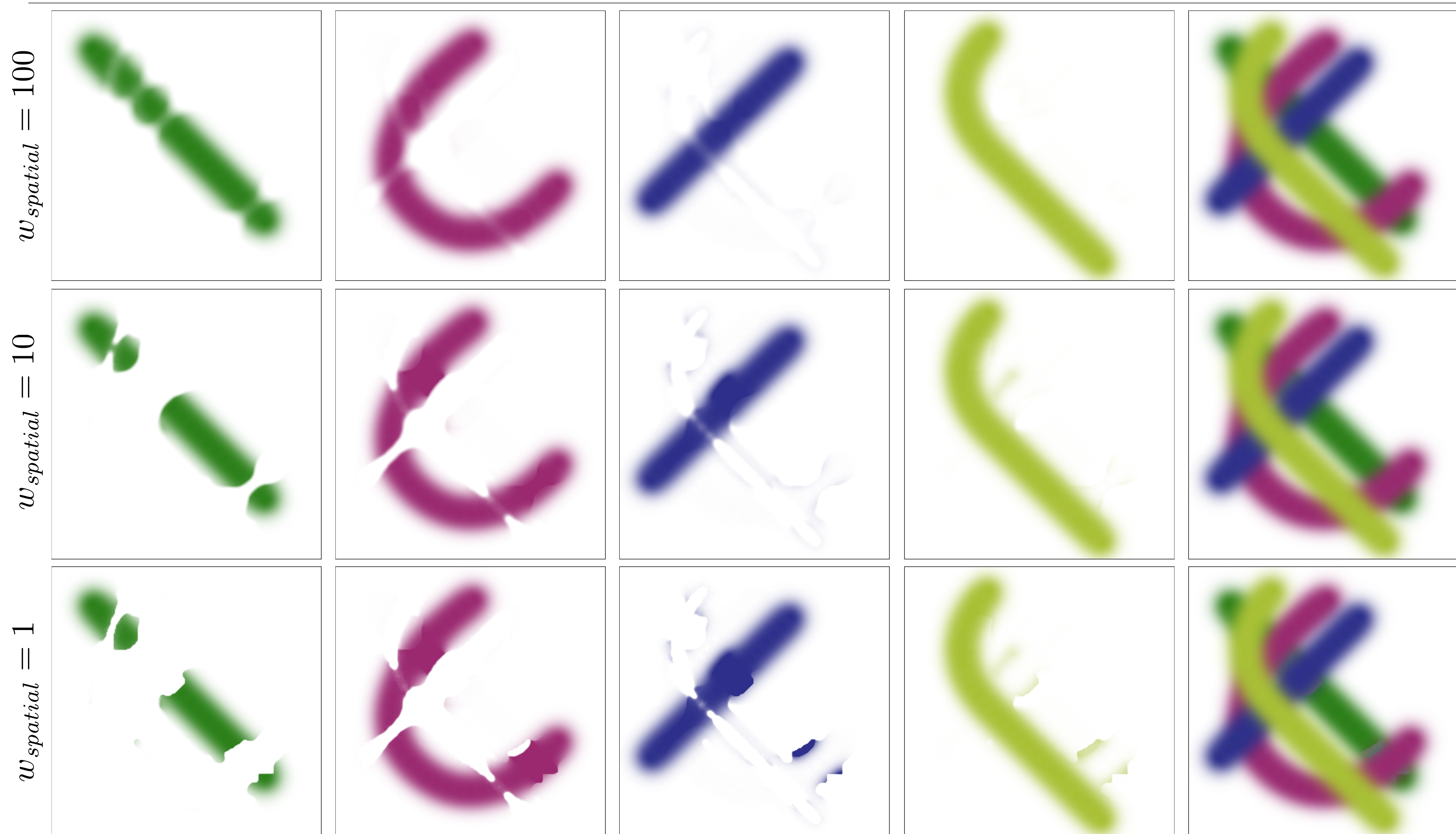
*adjusting  $w_{opaque}$*





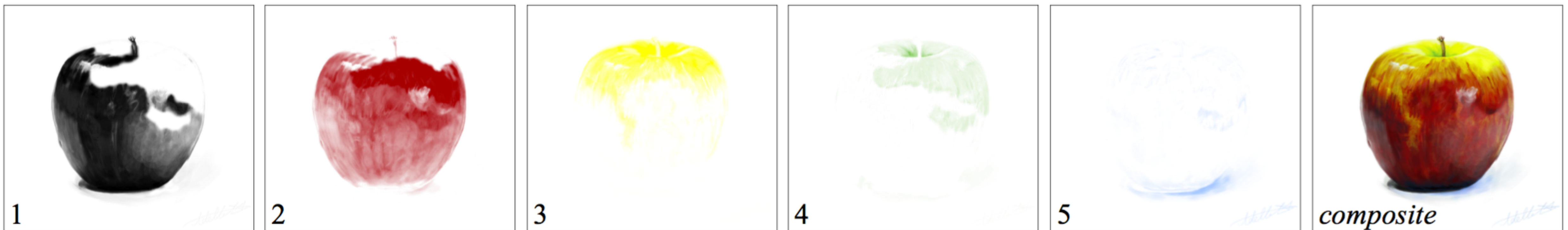
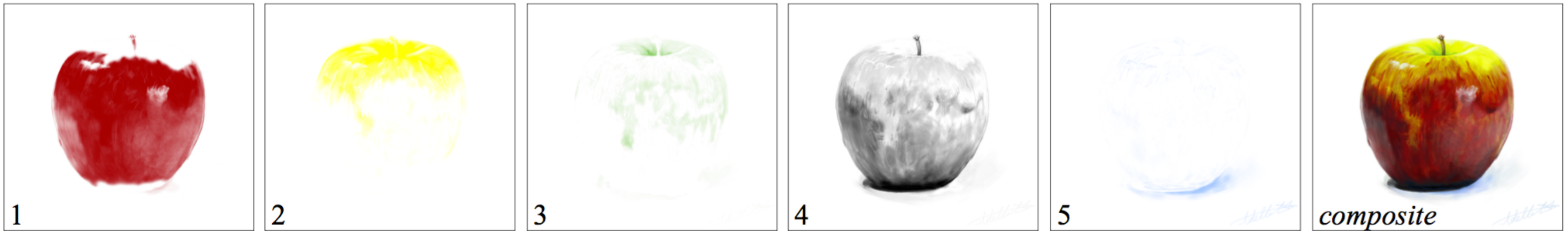
# optimization parameters influence

*adjusting  $w_{spatial}$*

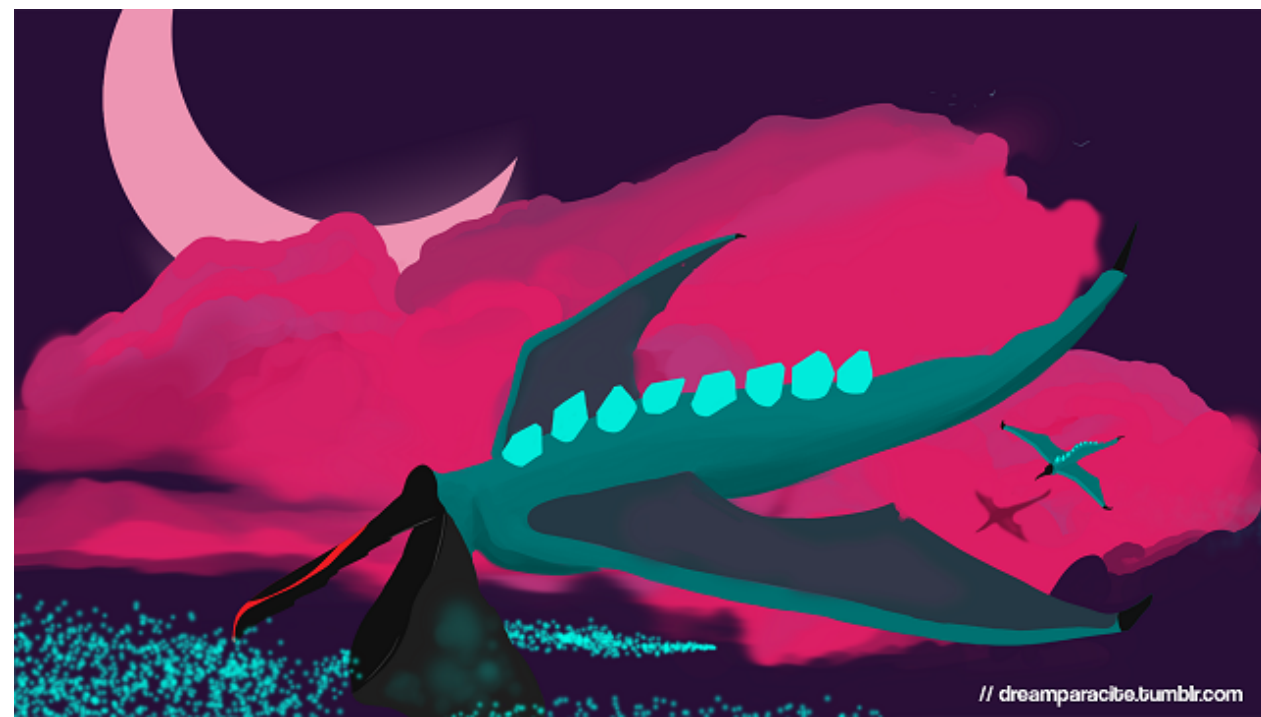




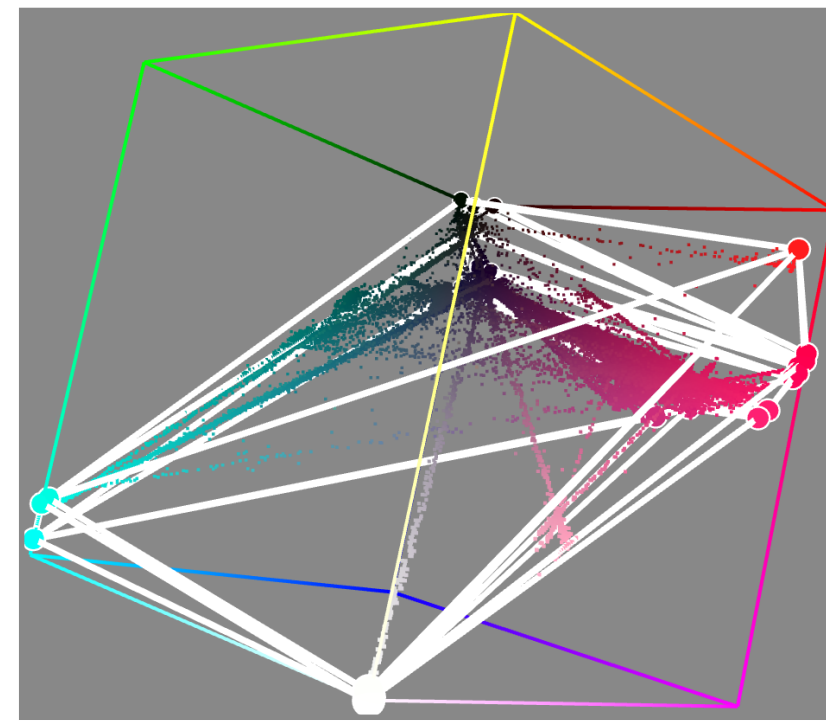
# Layer order influence



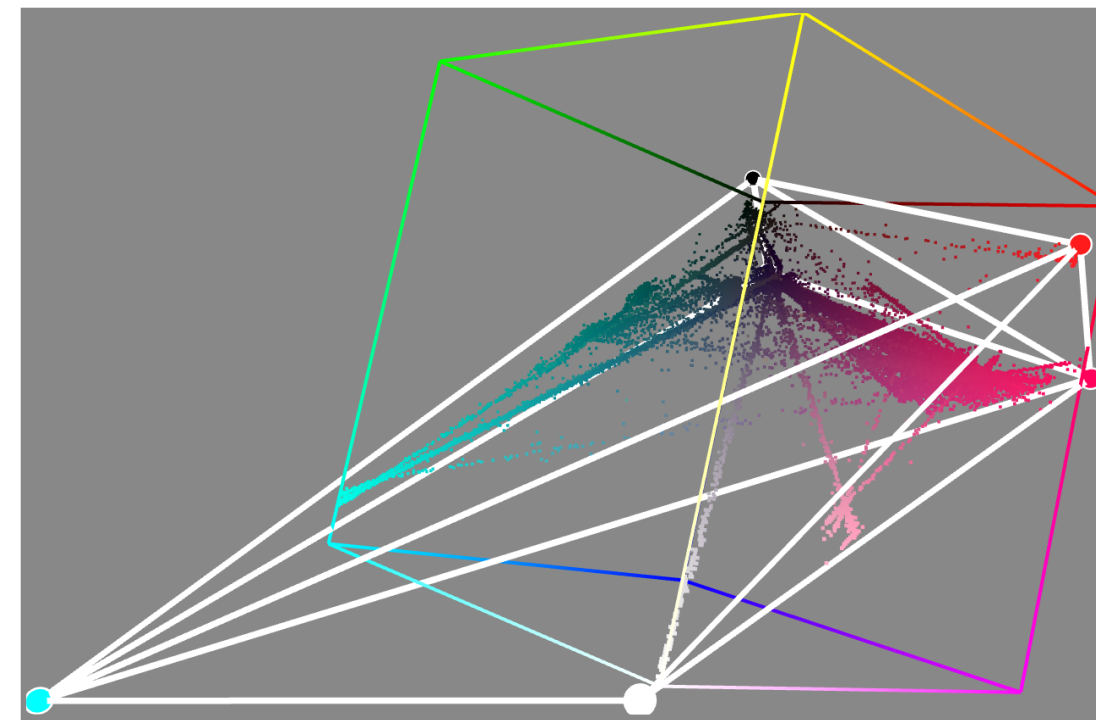
# Post vertex brute force optimization led to an improvement in vertex positions.



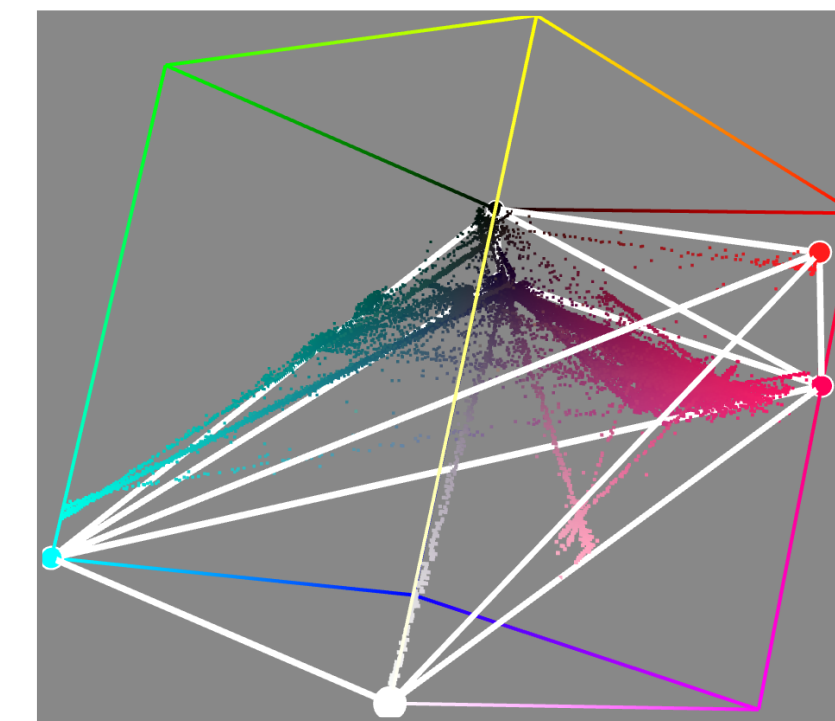
*input image*



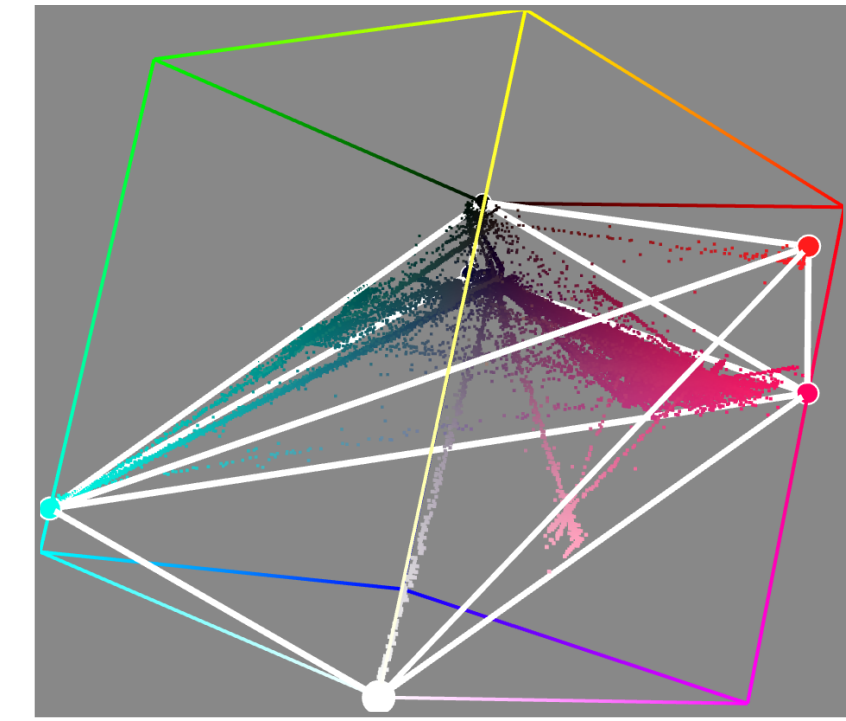
*convex hull*



*simplified hull with  
invalid colors*



*projected hull*



*optimized hull*



Extract additive mixing layers using our  
optimization

# Extract additive mixing layers using our optimization

$$\mathbf{E} = \begin{aligned} & \|\text{original} - \text{reconstructed image}\|^2 && \sum \|P_i - w_{ij}C_j\|^2 \\ & + && \\ & \text{Per pixel mixing weights sparsity} && \sum -(1 - w_{ij})^2 \\ & + && \\ & \text{Mixing weights spatial smoothness} && \text{(Laplacian)} \end{aligned}$$

# Spectral Matting [Levin et al. 2008]

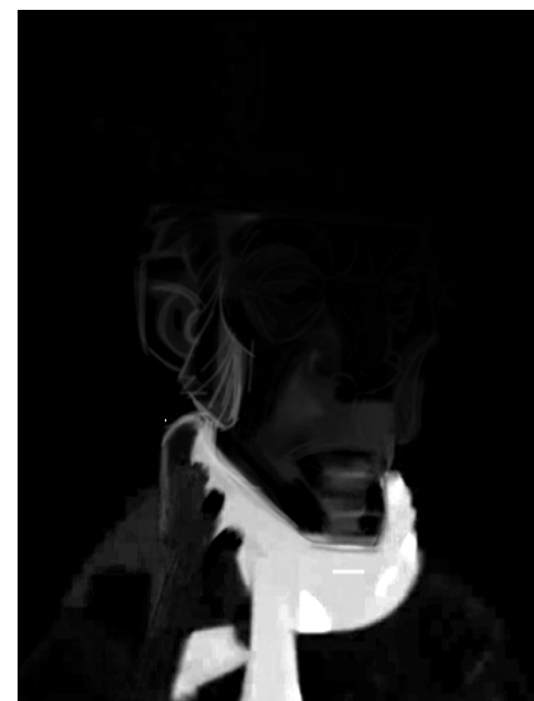
*input*



*9 components*



*1*



*2*



*3*



*4*



*5*



*6*



*7*



*8*



*9*



*k-means*

*best alpha matte*

